Accurate modeling and simulation of the dynamics of ultrashort optical pulses in nonlinear waveguides

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Outline

- Part 1 - Physics of the nonlinear Schrödinger equation (NSE) in fiber optics
- Part 2 - Modeling pulse propagation using the generalized NSE
- Part 3 - Two-frequency pulse compounds

Analytic signal based propagation models

- z-propagation of real-valued optical field
  - linearly polarized electromagnetic pulse
  - one-dimensional dispersive nonlinear medium
  - single-mode propagation
  - optical field:

$$E(z,t) = \mathsf{F}^{-1}\left[E_\omega(z)\right] = \sum_\omega E_\omega(z)\, e^{-i\omega t}, \quad \omega \in \frac{2\pi}{T}\mathbb{Z}$$

$$E_\omega(z) = \mathsf{F}\left[E(z,t)\right] = \frac{1}{T}\int_{-t_{\max}}^{t_{\max}} E(z,t)\, e^{i\omega t}\ \mathrm{d}t$$

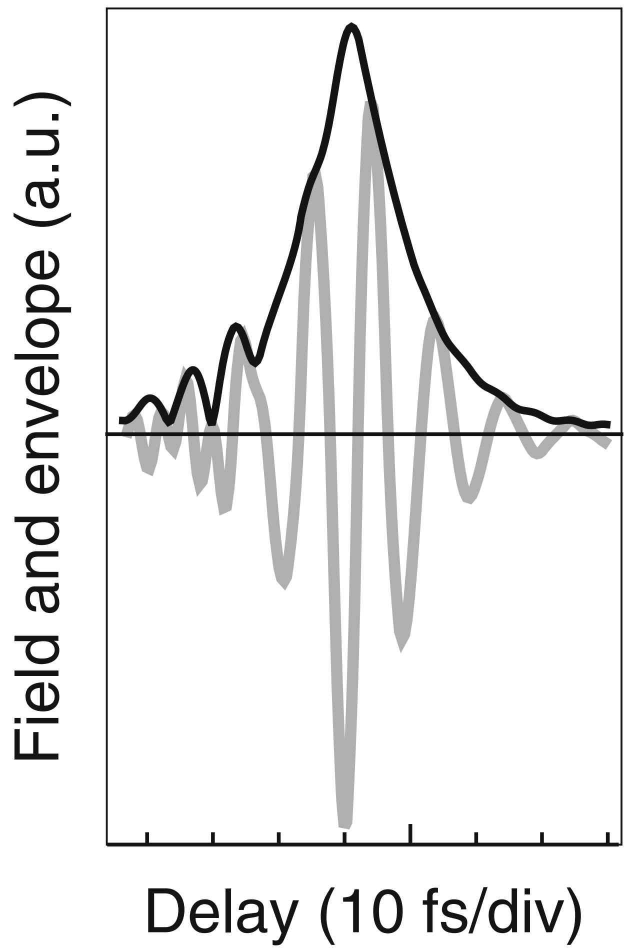


Figure taken from:
[Amiranashvili; in *New Approaches to Nonlin. Waves* (2016)]

- Forward model for the analytic signal

[Amiranashvili, Demircan; PRA 82 (2010) 013812]
[Amiranashvili, Demircan; AOT (2011) 989515]

$$i\partial_z \mathcal{E}_\omega + k(\omega)\mathcal{E}_\omega + \frac{3\omega^2\chi}{8c^2\beta(\omega)}\left(|\mathcal{E}|^2\mathcal{E}\right)_{\omega>0} = 0$$

- non-envelope model
  - ➡ spectrally broad pulses
  - ➡ ultrashort pulses

$\chi =$ nonlinear susceptibility
$c =$ speed of light
$\omega =$ angular frequency

- relation to optical field:

$$\mathcal{E}(z,t) = \sum_{\omega>0} \mathcal{E}_\omega(z)\, e^{i\omega t}, \quad \mathcal{E}_\omega(z) = \left[1 + \mathrm{sign}(\omega)\right] E_\omega(z)$$

- wavenumber:

$$k(\omega) = \beta(\omega) + i\alpha(\omega)$$

- conservation law ($\alpha(\omega) = 0$):

$$C_p(z) = \sum_{\omega>0} \omega^{-2}\,\beta(\omega)\,|\mathcal{E}_\omega(z)|^2$$

(classical analog of photon number)

Analytic signal based propagation models

- Equivalence to nonlinear Schrödinger equation in SVEA* limit

  - simplify wavenumber

$$\alpha(\omega) = 0,\ \beta(\omega = \omega_0 + \Omega) = \beta_0 + \beta_1\Omega + \frac{\beta_2}{2}\Omega^2$$

  - introduce reference frequency and shift to moving frame of reference

$$A(z,\tau) = \sum_{\Omega} A_\Omega(z) e^{-i\Omega\tau}, \quad A_\Omega(z) = \mathcal{E}_{\omega_0+\Omega}(z) e^{-i(\beta_0+\beta_1\Omega)z}, \quad \tau = t - \beta_1 z$$

  - simplify nonlinearity

$$\gamma = \frac{3\,\omega_0\,\chi}{8\,c\,n(\omega_0)}$$

  - rewrite as standard nonlinear Schrödinger equation (NSE)

$$i\partial_z A_\Omega + \frac{\beta_2}{2}\Omega^2 A_\Omega + \gamma\left(|A|^2 A\right)_\Omega = 0$$

(Frequency domain representation)

$$i\partial_z A - \frac{\beta_2}{2}\,\partial_\tau^2 A + \gamma\,|A|^2 A = 0$$

(Time domain representation)

  - selected conservation law

$$C_E(z) = \int_{-\infty}^{\infty} |A(z,t)|^2\ \mathrm{d}\tau$$

[Zhakarov, Shabat; JETP 34 (1972) 62]

* Slowly varying envelope approximation (SVEA)

Part 1

Physics of the 1D NSE in fiber optics

1D NSE in fiber optics notation

$$i\partial_z A = \frac{\beta_2}{2}\partial_\tau^2 A - \gamma |A|^2 A$$

$A = A(z,t) = \text{slowly varying pulse envelope}$

$\gamma = \text{nonlinear parameter (W}^{-1}\text{/km)}$

$\tau = t - \beta_1 z = \text{retarded time (ps)}$

$\beta_1 = 1/v_g = \text{group delay (ps/km)}$

$\beta_2 = \text{group-velocity dispersion (ps}^2\text{/km)}$

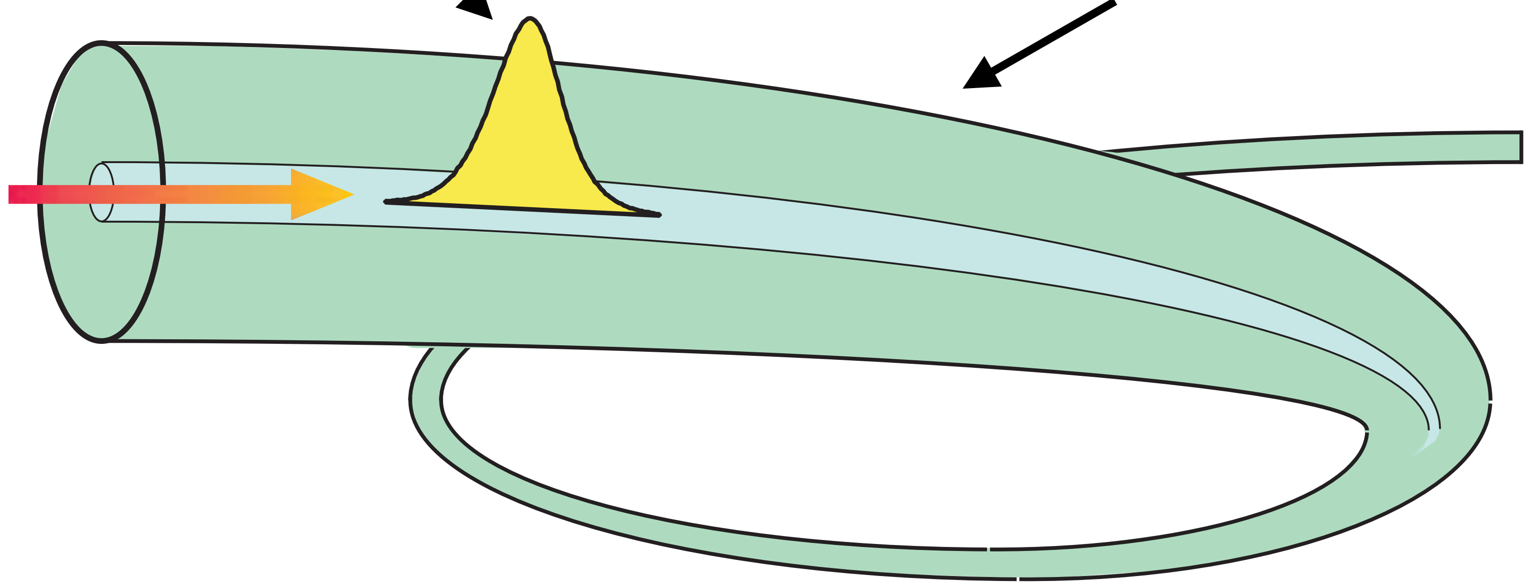
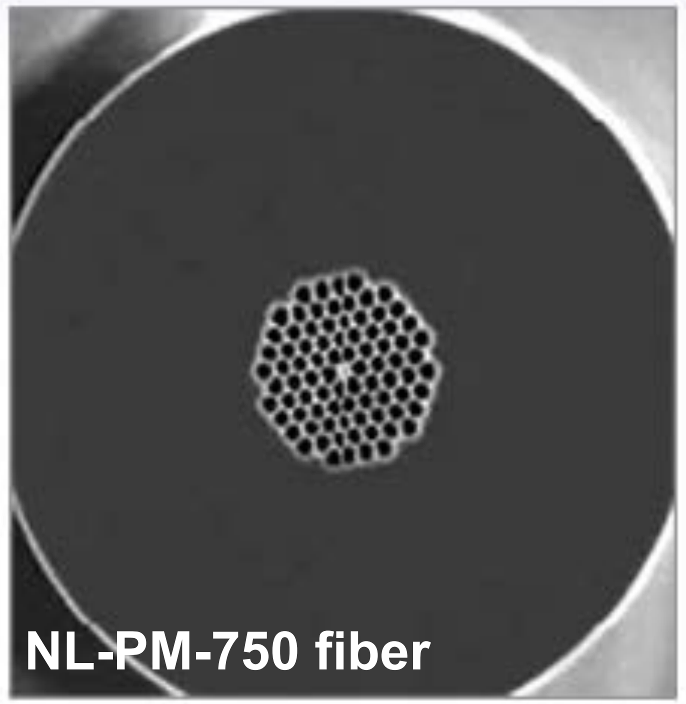


Figure taken from:
[Philbin *et al.*; Science 319 (2008) 1367]

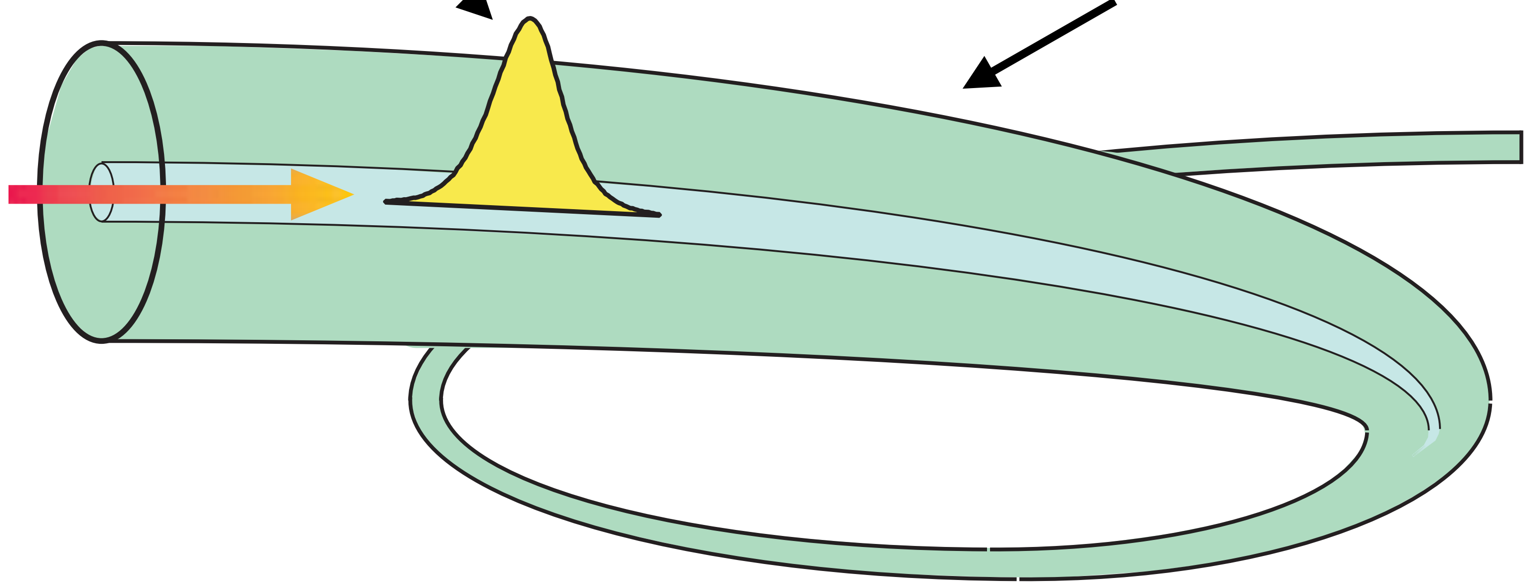
- exactly integrable partial differential equation (PDE)
  ➡ obeys infinitely many conservation laws
  [Zhakarov, Shabat; JETP 34 (1972) 62]
- describes nonlinear propagation of waves
  ➡ applies to fluids, optics, Bose-Einstein condensates
  [Yang; *Nonlinear waves in integrable and nonintegrable systems* (2010)]
- can be solved using the inverse scattering transform
  ➡ provides exact solutions known as solitons
  [Agrawal; *Nonlinear Fiber Optics* (2019)]



Core diameter: $1.8 - 3.2\ \mu\text{m}$

1D NSE in fiber optics notation

$$i\partial_z A = \frac{\beta_2}{2}\partial_\tau^2 A - \gamma|A|^2 A$$



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$A = A(z,t) =$ slowly varying pulse envelope

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$\tau = t - \beta_1 z =$ retarded time (ps)

$\beta_1 = 1/v_g =$ group delay (ps/km)

$\beta_2 =$ group-velocity dispersion $(\mathrm{ps}^2/\mathrm{km})$

- Split-step Fourier method (SSFM)
  - nonlinear term *easily* evaluated in time-domain
  - derivatives *easily* evaluated in Fourier domain

    $$\partial_\tau^n \longrightarrow (-i\Omega)^n, \quad \partial_\tau^2 A \longrightarrow -\Omega^2 A_\Omega$$

  - simple approximate solution procedure
    [Taha, Ablowitz; J. Comp. Phys. 55 (1984) 203]

$$\xi = \exp\{i\gamma|A(z,t)|^2\Delta z\}\, A(z,t)$$

$$A(z+\Delta z, t) = \mathsf{F}^{-1}\left[\exp\{i(\beta_2/2)\Omega^2\Delta z\}\,\mathsf{F}[\xi]\right]$$

➡ simple but not recommended; global error $\mathcal{O}(\Delta z)$

- Popular fixed stepsize method

  4th order Runge-Kutta in the interaction picture method
  [Hult; IEEE J. Lightwave Tech. 25 (2007) 3770]

- Tailored adaptive stepsize methods

  LEM: Local error method
  [Sinkin *et al.*; IEEE J. Lightwave Tech. 21 (2003) 61]

  CQE: Conservation quantity error method
  [Heidt; IEEE J. Lightwave Tech. 27 (2009) 3984]

  B43: Balac 4(3) ERK method
  [Balac, Mahe; Comp. Phys. Commun. 184 (2013) 1211]

Rich variety of dynamical phenomena - Solitons

- Optical temporal solitons
  - exist for anomalous dispersion $\beta_2 < 0$
  - evolve without change in shape and spectrum
    - ➡ balance of dispersion and nonlinearity
  - *localized* in time, *stationary* along z
    - ➡ temporal solitons

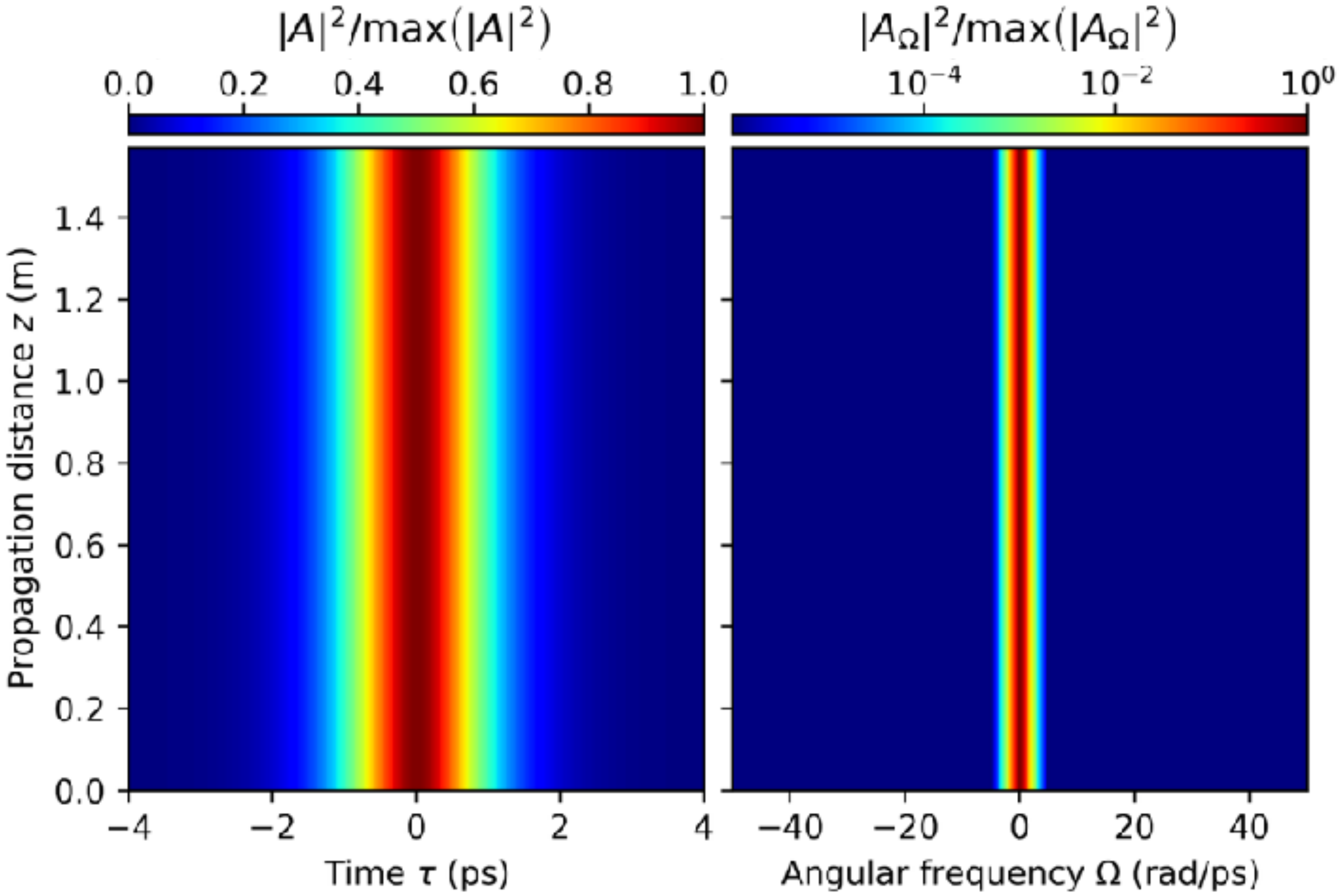
- Fundamental soliton

$$A(z,\tau) = A_0 \,\mathrm{sech}\left(\frac{\tau}{t_0}\right)\, e^{i\frac{\gamma P_0}{2} z}$$

$$P_0 = A_0^2 = \frac{|\beta_2|}{\gamma t_0^2}$$

  - dispersion length: $L_D = t_0^2/|\beta_2|$
  - nonlinear length: $L_{\mathrm{NL}} = (\gamma P_0)^{-1}$
  - soliton energy: $E = 2\,t_0\,P_0$

$$\frac{L_D}{L_{\mathrm{NL}}} = 1$$



- Prediction + demonstration of fiber-optical solitons

[Hasegawa, Tappert; Appl. Phys. Lett. 23 (1973) 142]

[Mollenauer, Stolen, Gordon; PRL 45 (1980) 1095]

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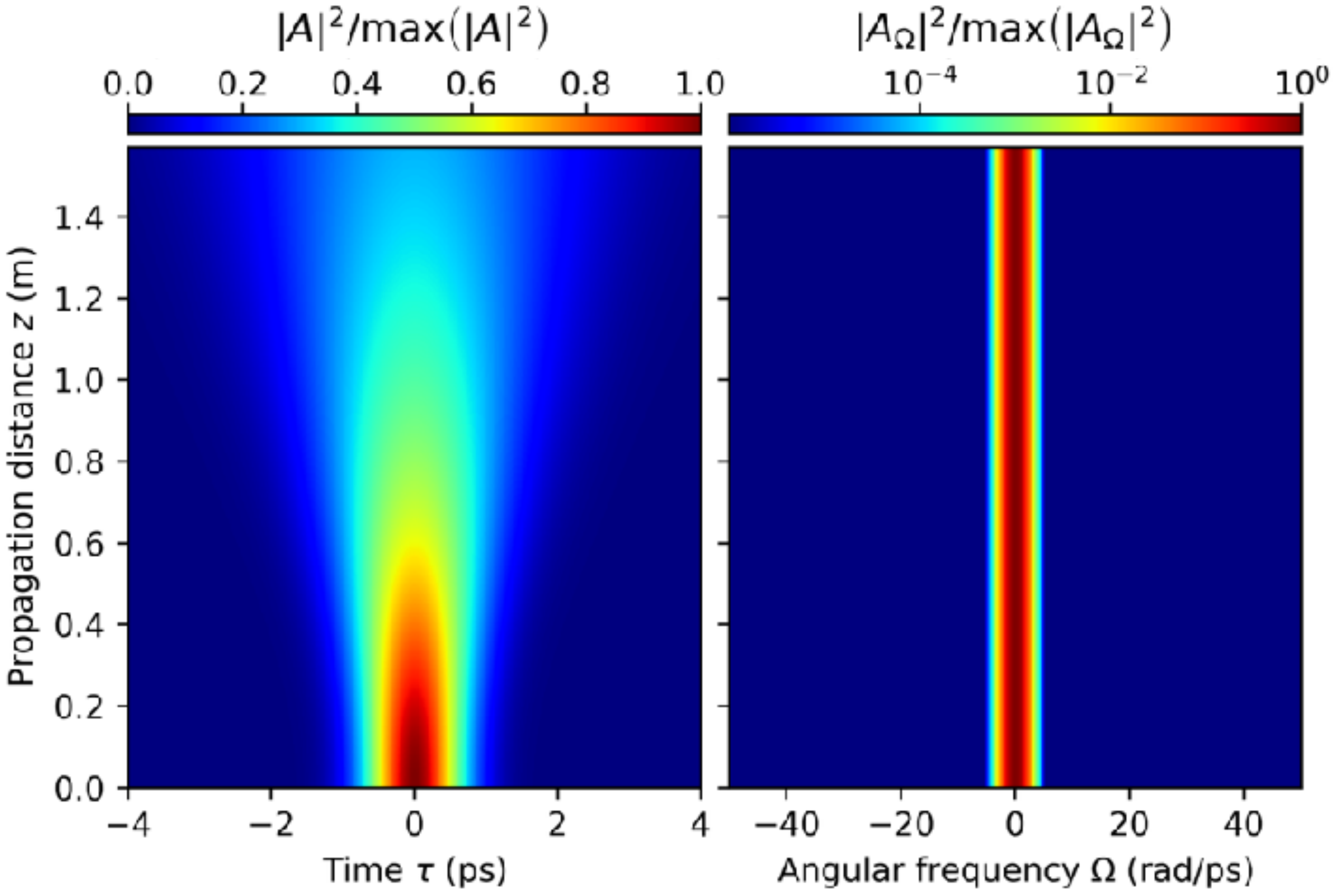
$$A(0,\tau) \propto \exp\{-(\tau/t_0)^2\}$$



- Non-soliton regimes (for comparison)
  - dispersion-dominant

$$\frac{L_D}{L_{\mathrm{NL}}} \ll 1$$

Rich variety of dynamical phenomena - Solitons

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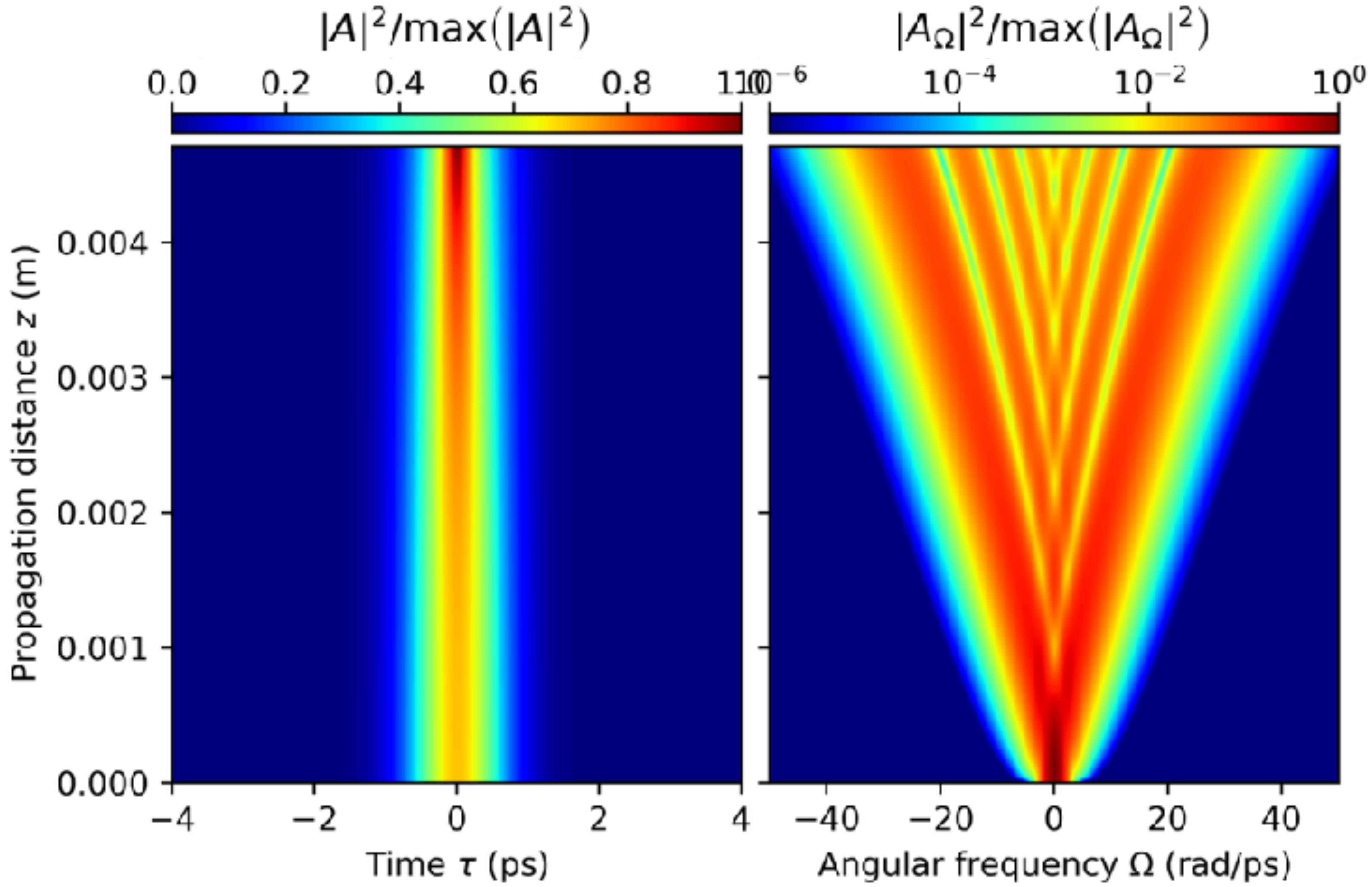


- Non-soliton regimes (for comparison)
  - nonlinearity-dominant
  - self-phase modulation

$$\frac{L_D}{L_{\mathrm{NL}}} \gg 1$$

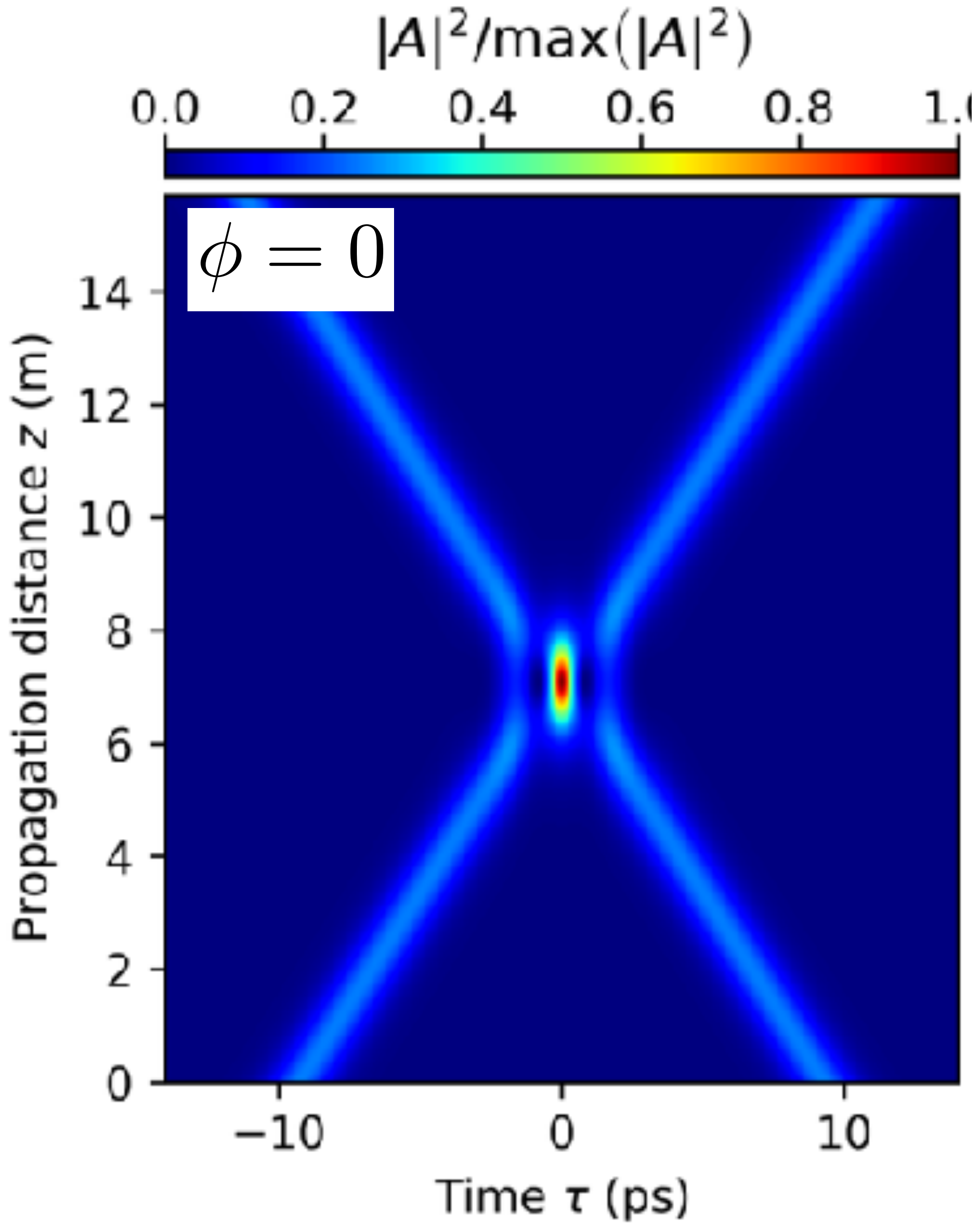
PhoeniXD

Interactions between solitons

- NSE solitons collide elastically
  - exhibit particle-like properties
  - coherent interaction
    - ➡ affected by relative phase

- Initial condition for colliding solitons

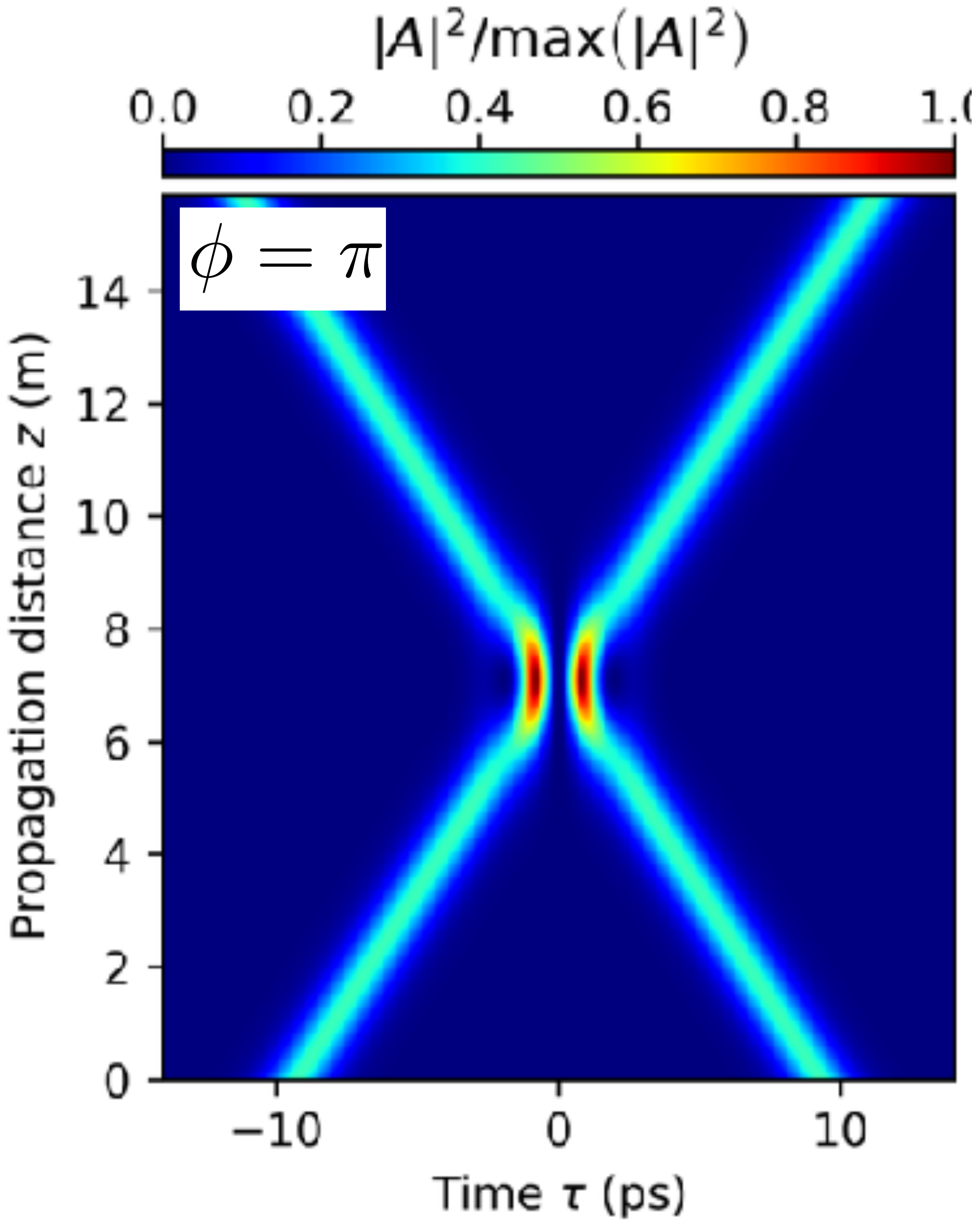
$$A(0,t) = A_0 \,\mathrm{sech}\left(\frac{t-\delta}{t_0}\right) e^{i(\omega_0 t+\phi)} + A_0 \,\mathrm{sech}\left(\frac{t+\delta}{t_0}\right) e^{-i\omega_0 t}$$



- Collisions for NSE solitons
  - number of solitons is conserved
  - no energy lost to radiation
  - velocities don't change
  - transient spectral shift
  - imprints phase and time shift

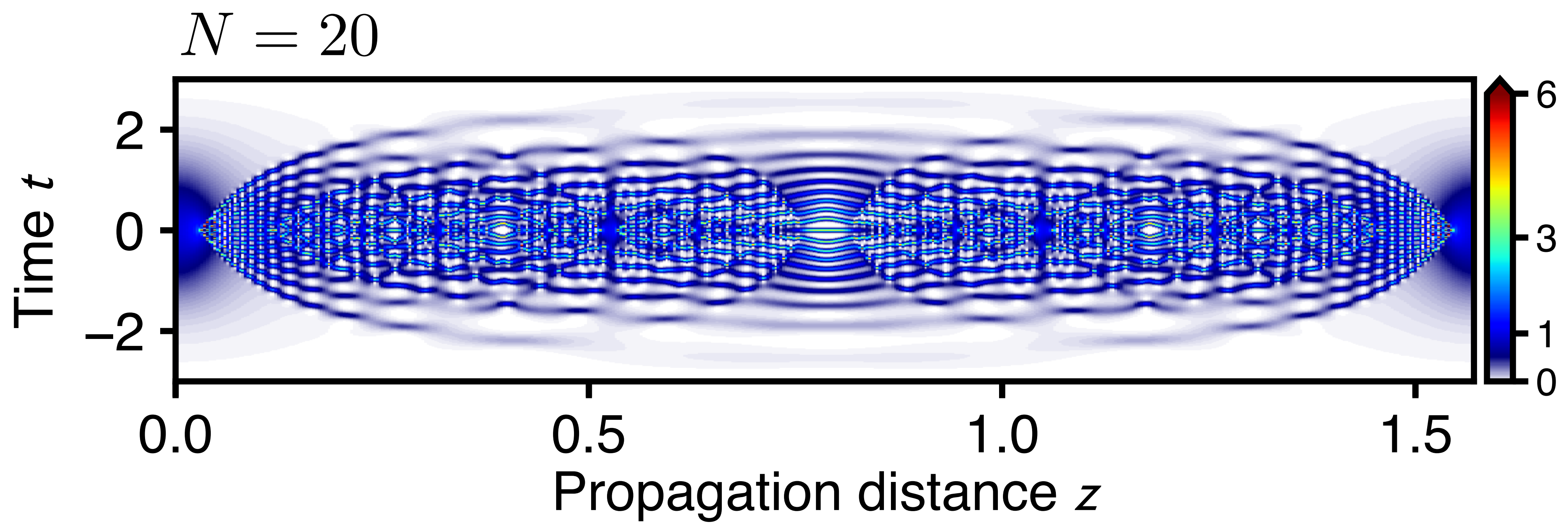
⬅ solitons in phase

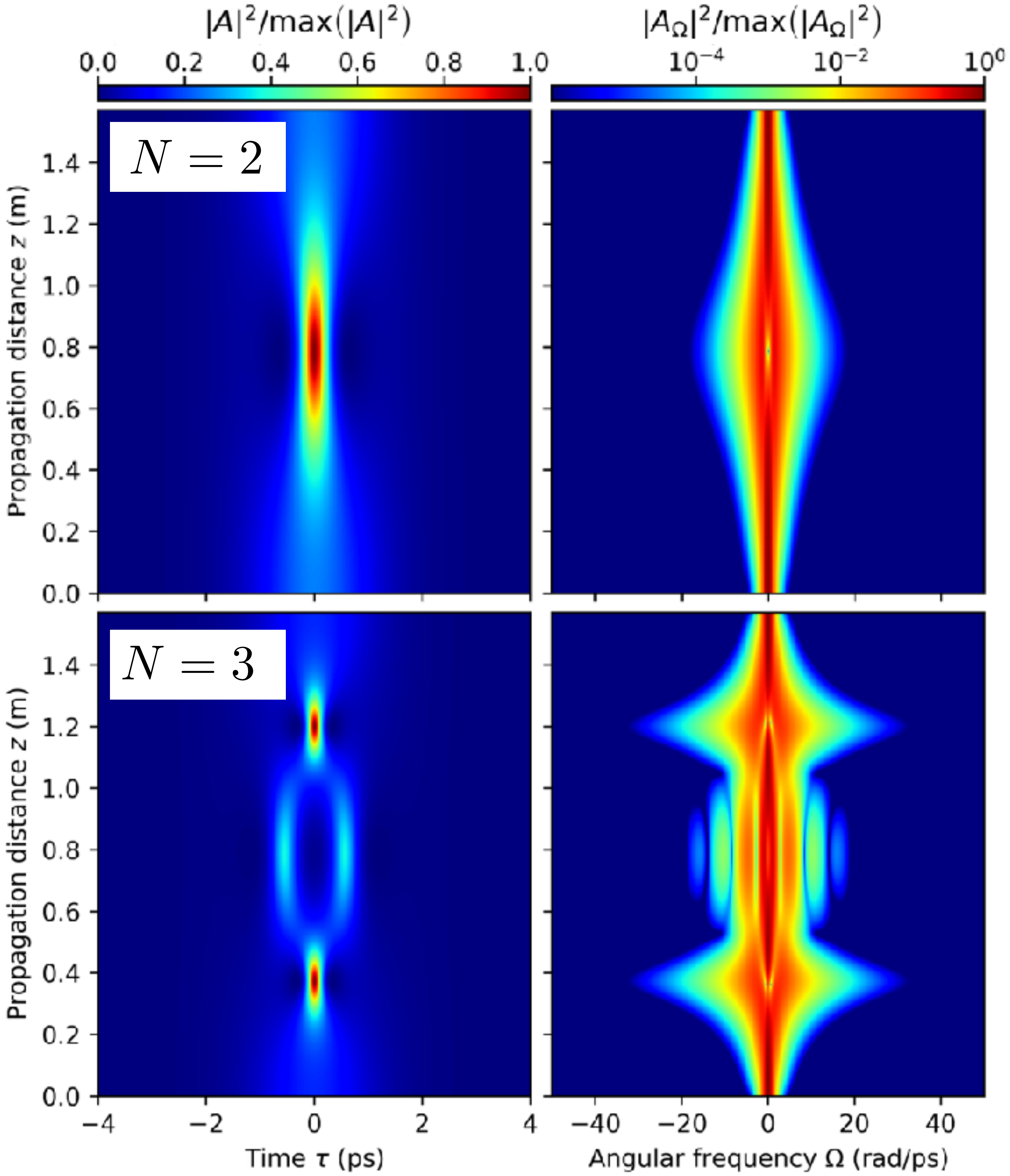
solitons in antiphase ➡



Higher-order solitons

- Higher-order solitons
  - Bound-state of N solitons
  - *localized* in time, *periodic* along z
  - amplitude: $A_0^{\mathrm{N-sol}} = N\,A_0, \quad N^2 = \dfrac{L_D}{L_{\mathrm{NL}}}$
  - soliton period: $z_s = \dfrac{\pi}{2} L_D$
  - correct propagation for large N requires high accuracy
    ➡ tough test for numerical algorithms





Third-order dispersion

- NSE perturbed by third-order dispersion

$$i\partial_z A = \left(\frac{\beta_2}{2}\partial_\tau^2 - i\frac{\beta_3}{6}\partial_\tau^3\right) A - \gamma|A|^2 A$$

$$k_{\mathrm{lin}}(\Omega) = \frac{\beta_2}{2}\Omega^2 + \frac{\beta_3}{6}\Omega^3$$

$$\beta_3 = \text{third-order dispersion (ps}^2/\text{km)}$$

  - describes dynamics for zero-dispersion points

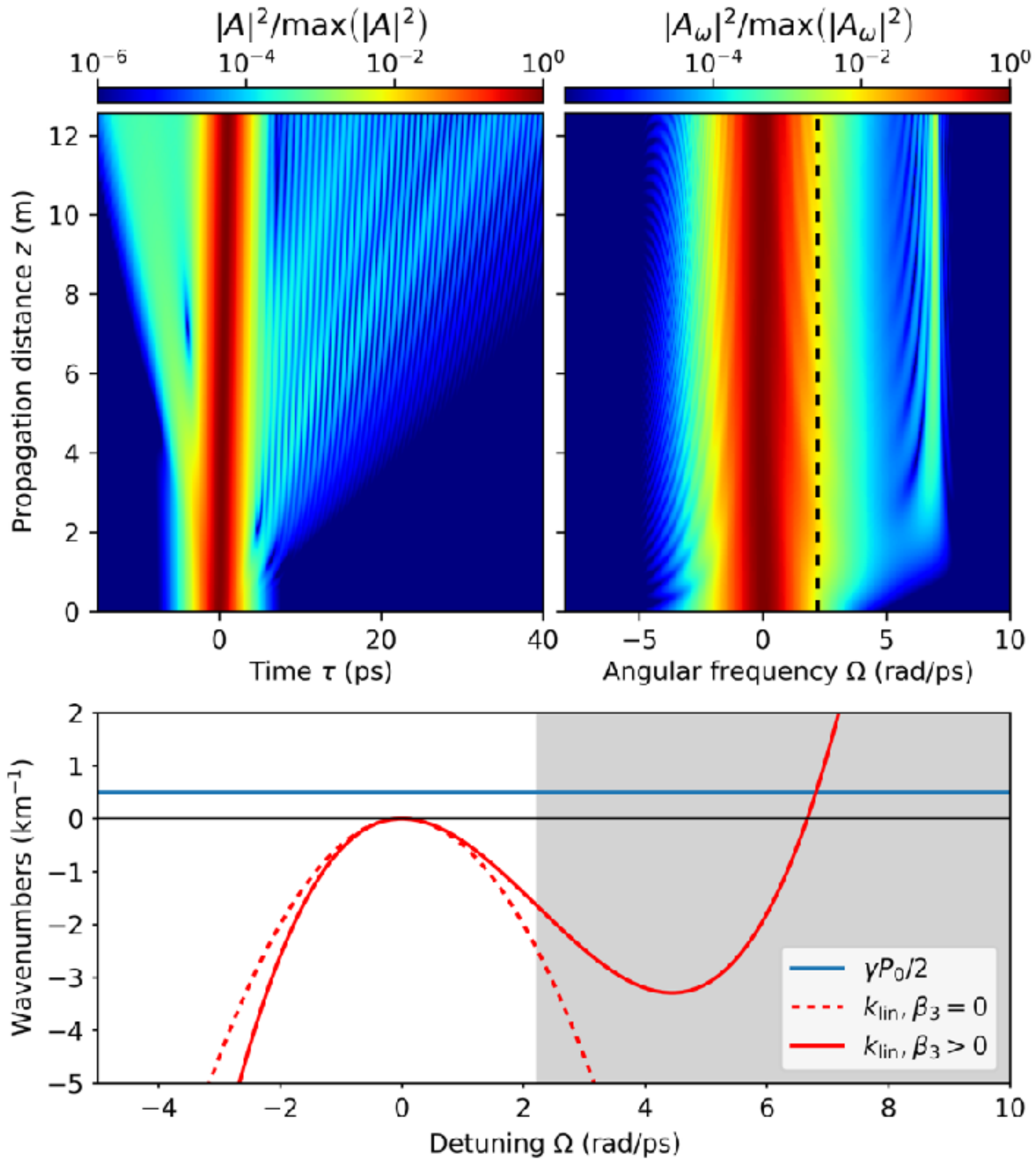
$$\partial_\Omega^2 k_{\mathrm{lin}}(\Omega_Z) \overset{!}{=} 0 \;\rightarrow\; \Omega_Z = -\frac{\beta_2}{\beta_3}$$

- Emission of resonant radiation
  - radiation frequency

$$k_{\mathrm{lin}}(\Omega_{\mathrm{RR}}) = \frac{\gamma P_0}{2}$$

  ➡ optical Cherenkov radiation

[Akhmediev, Karlsson; 51 (1995) 2602]
[Skryabin, Yulin; PRE 72 (2005) 016619]



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Interaction of pulses across a zero-dispersion point

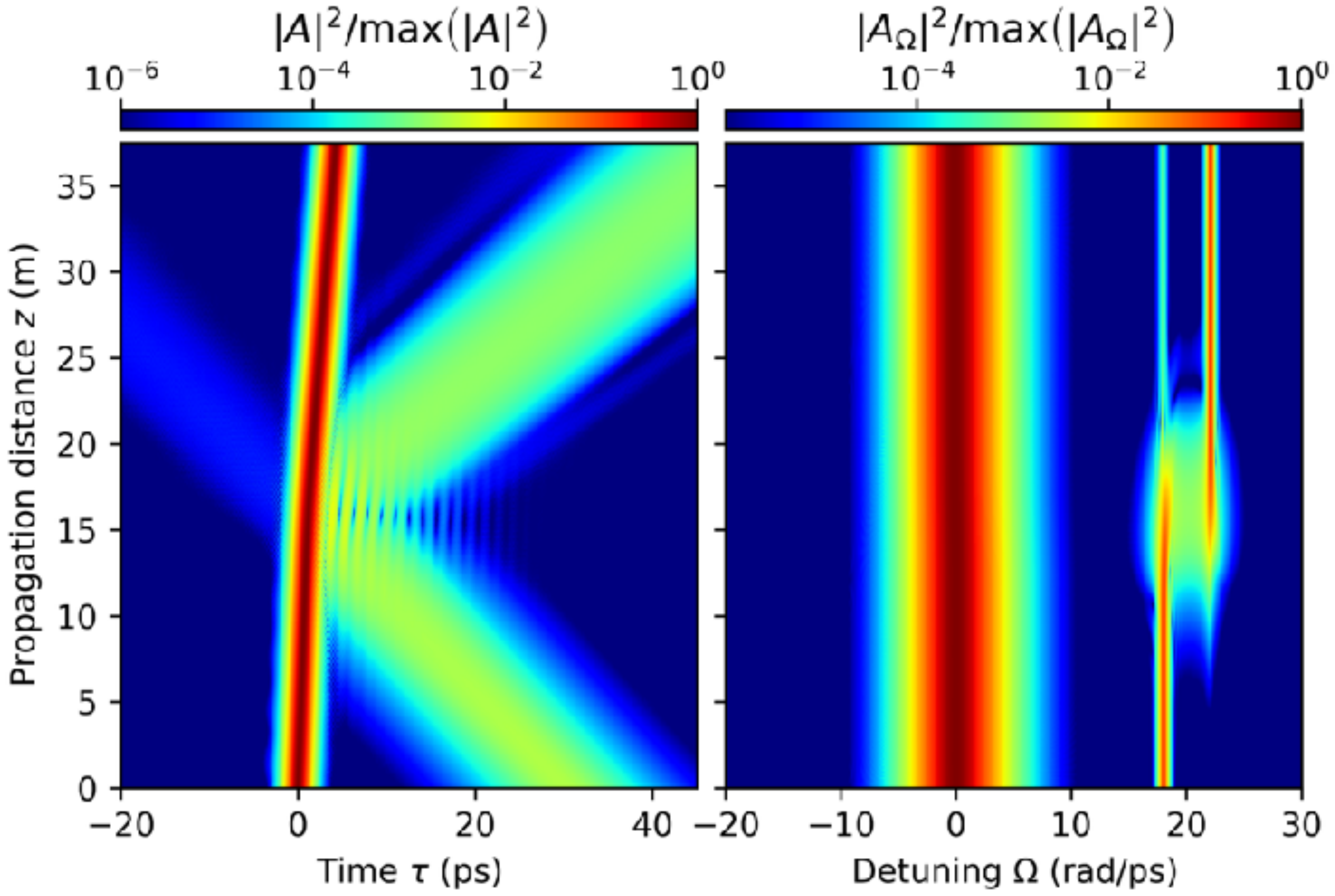
- Interaction between soliton (S) and dispersive wave (DW)
  - co-propagation with similar group velocity
  - strong *repulsive* interaction
  - based on general wave reflection mechanism

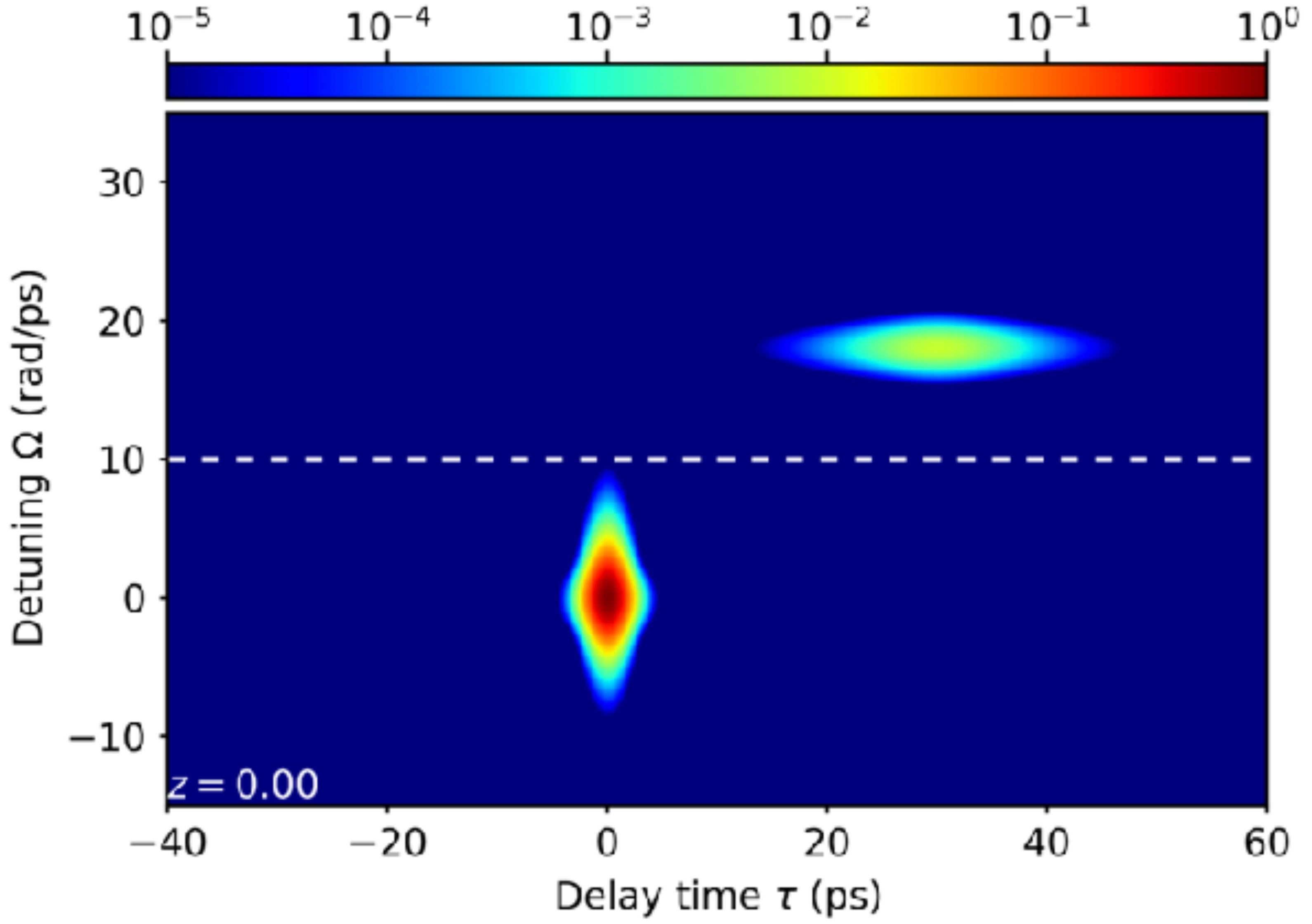
[Smith, Math. Proc. Camb. Phil. Soc 78 (1975) 517]
[de Sterke, Opt. Lett. 17 (1992) 914]
[Philbin *et al.*, Science 319 (2008) 1367]
[Demircan *et al.*, PRL 106 (2011) 163901]
[Faccio, Cont. Phys. 1 (2012) 1]

- Frequency shifts in presence of (almost) stationary solitons

$$P_S(z,\tau,\Omega) = \left| \int A(z,t)h(t-\tau)e^{-i\Omega t}\,\mathrm{d}t \right|^2$$

$$h(x) = \exp\{-x^2/2\sigma^2\},\ \sigma = 1\ \mathrm{ps}$$





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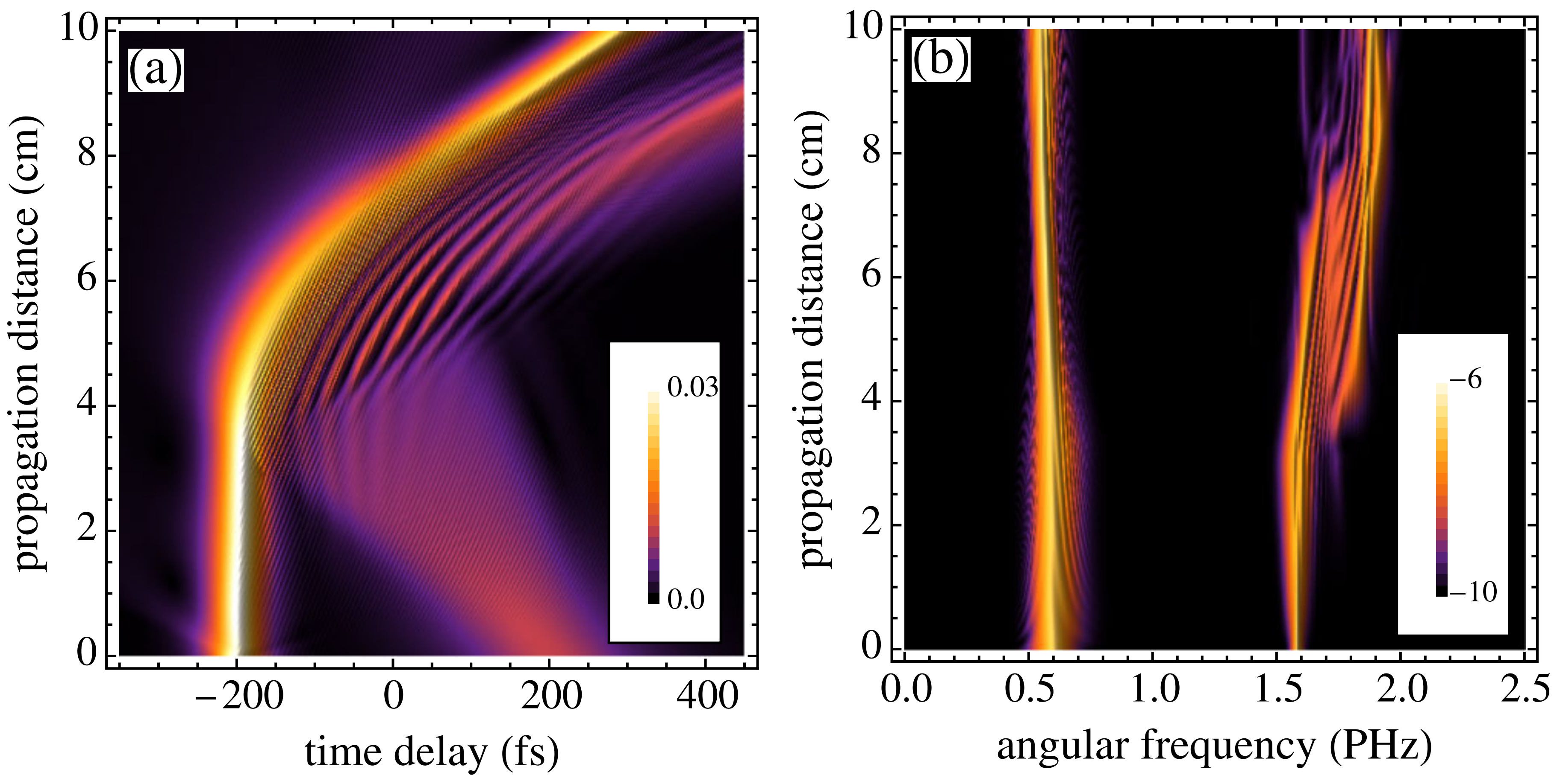
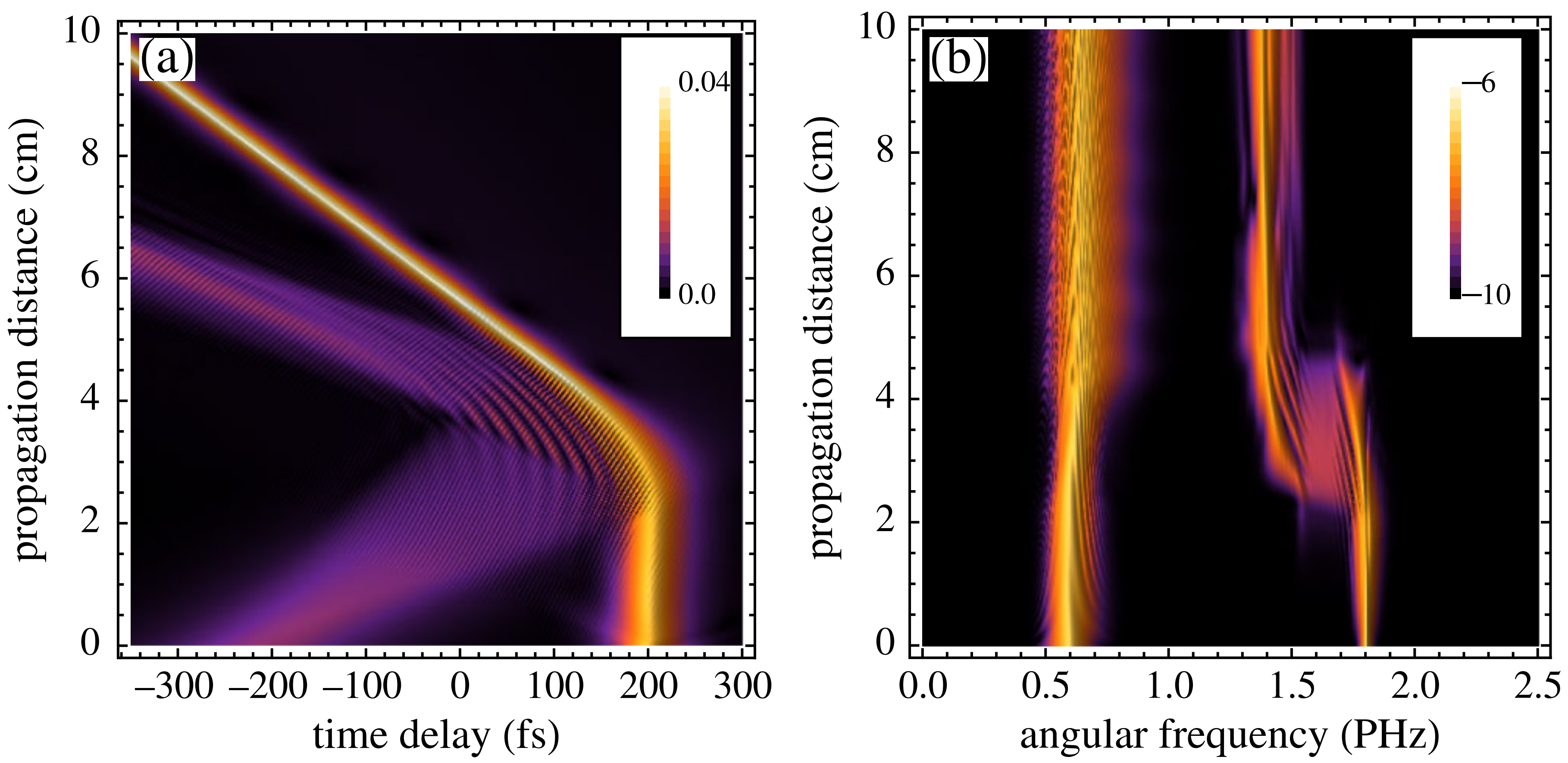
[Faccio, Cont. Phys. 1 (2012) 1]

- Strong + efficient light-light interaction (**here:** beyond the standard NSE model)
  [Demircan et al., PRL 106 (2011) 163901]
  - energy transfer from S to DW
  - energy transfer from DW to S





Part 2

Modeling pulse propagation using the generalized NSE

Generalized nonlinear Schrödinger equation (GNSE)

$$\partial_z A(z,t) = i \sum_{k\geq 2}^{11} \frac{\beta_k}{k!} (i\partial_t)^k A(z,t) + i\gamma \left(1 + \frac{1}{\omega_0} i\partial_t\right) A(z,t) \int_{-\infty}^{\infty} h(t') |A(z,t-t')|^2 \, \mathrm{d}t'$$

field envelope — dispersion operator — self-steepening — total response function

- Generalized nonlinear Schrödinger equation (GNSE)
  [Dudley, Genty, Coen; Rev. Mod. Phys. 78 (2006) 1135]
  - Applicable beyond slowly varying envelope approximation
    [Brabec, Krausz; Phys. Rev. Lett. 78 (1997) 3282]
  - Includes instantaneous Kerr and delayed Raman response
    [Blow, Wood; IEEE J. Quant. Electr. 25 (1989) 2665]

$$h(t) = (1 - f_R)\delta(t) + f_R h_R(t) \qquad f_R = 0.18$$

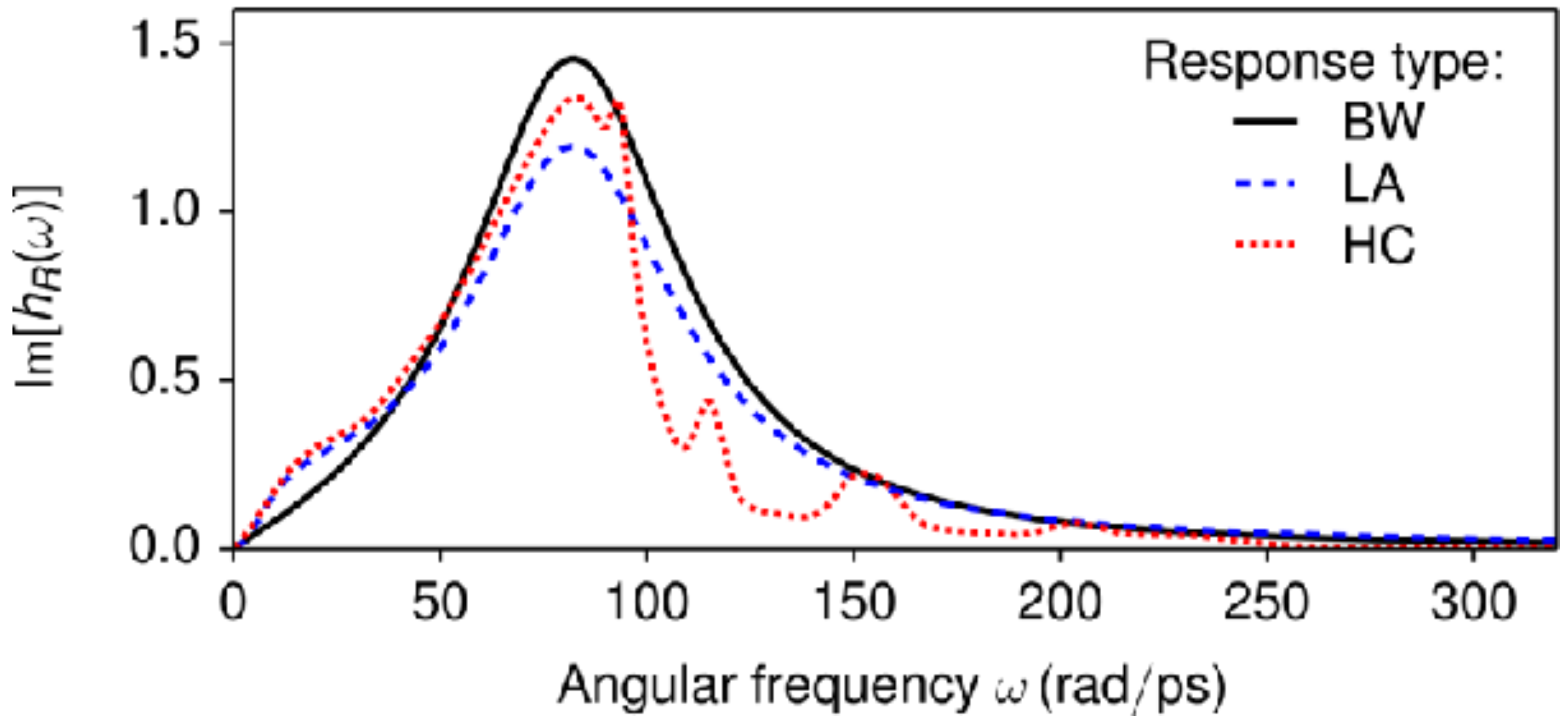
$$h_R(t) = \frac{\tau_1^2 + \tau_2^2}{\tau_1 \tau_2^2} e^{-t/\tau_2} \sin(t/\tau_1)\theta(t) \qquad \tau_1 = 12.2 \text{ fs}, \quad \tau_2 = 32.0 \text{ fs}$$

  - Conservation law (classical analog of photon number)

$$\partial_z \int \frac{|A_\omega(z)|^2}{\omega_0 + \omega} \, \mathrm{d}\omega = 0$$

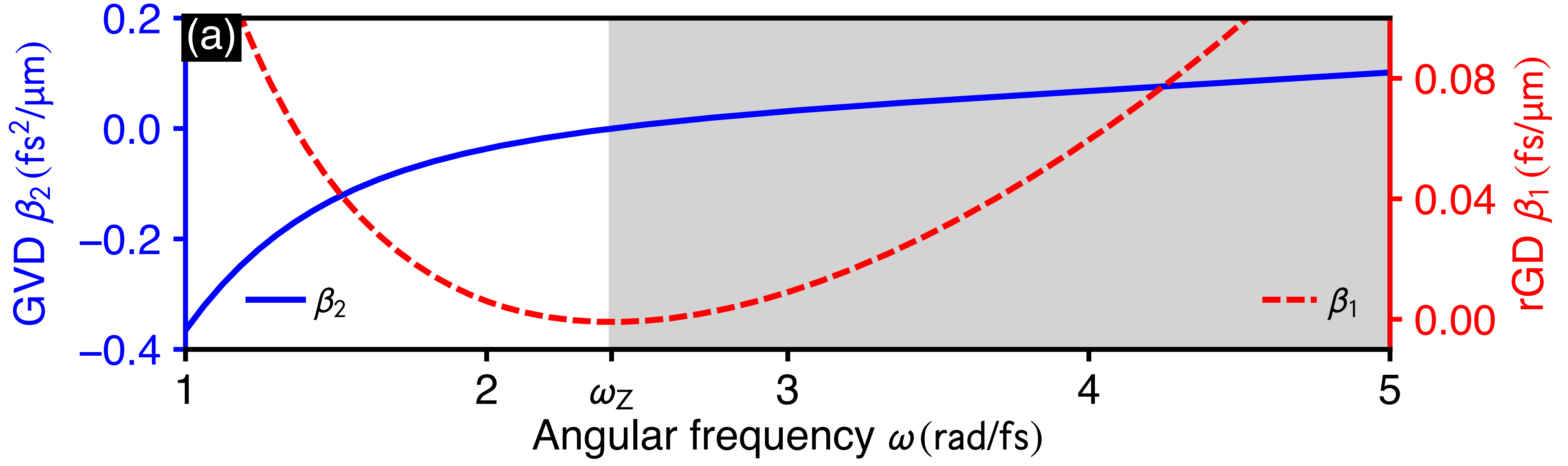
- Raman response models for silica fibers

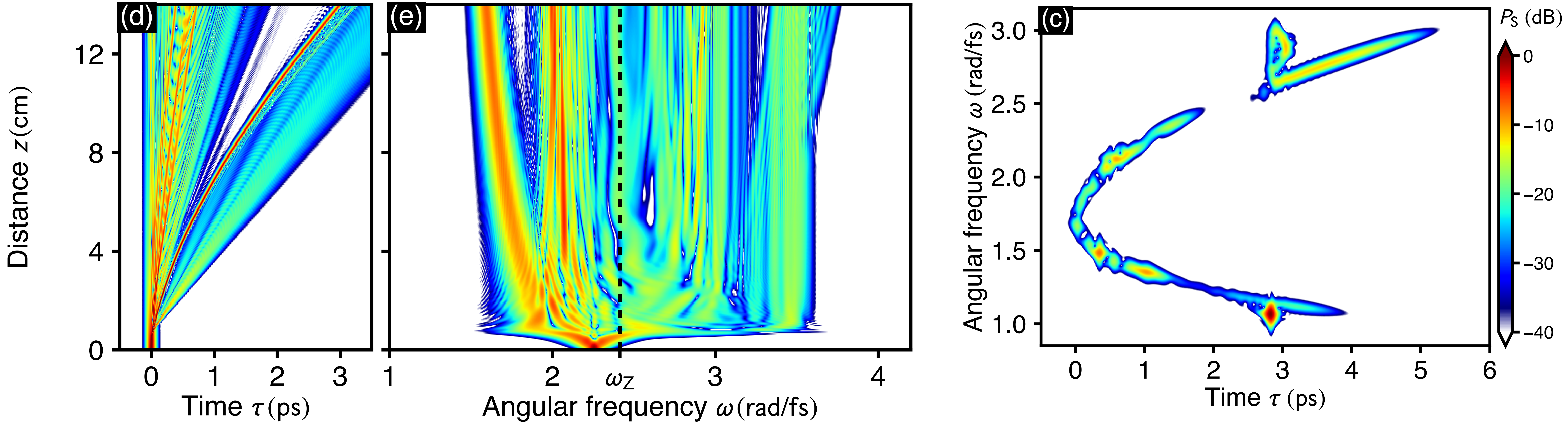
BW: **[Blow, Wood; IEEE J. Quant. Electron. 25 (1989) 2665]**
LA: **[Lin, Agrawal; Opt. Lett. 21 (2006) 3086]**
HC: **[Hollenbeck, Cantrell; JOSA B 19 (2002) 2886]**



Supercontinuum generation

- Effects leading to extreme spectral broadening
  - soliton fission + soliton self-frequency shift

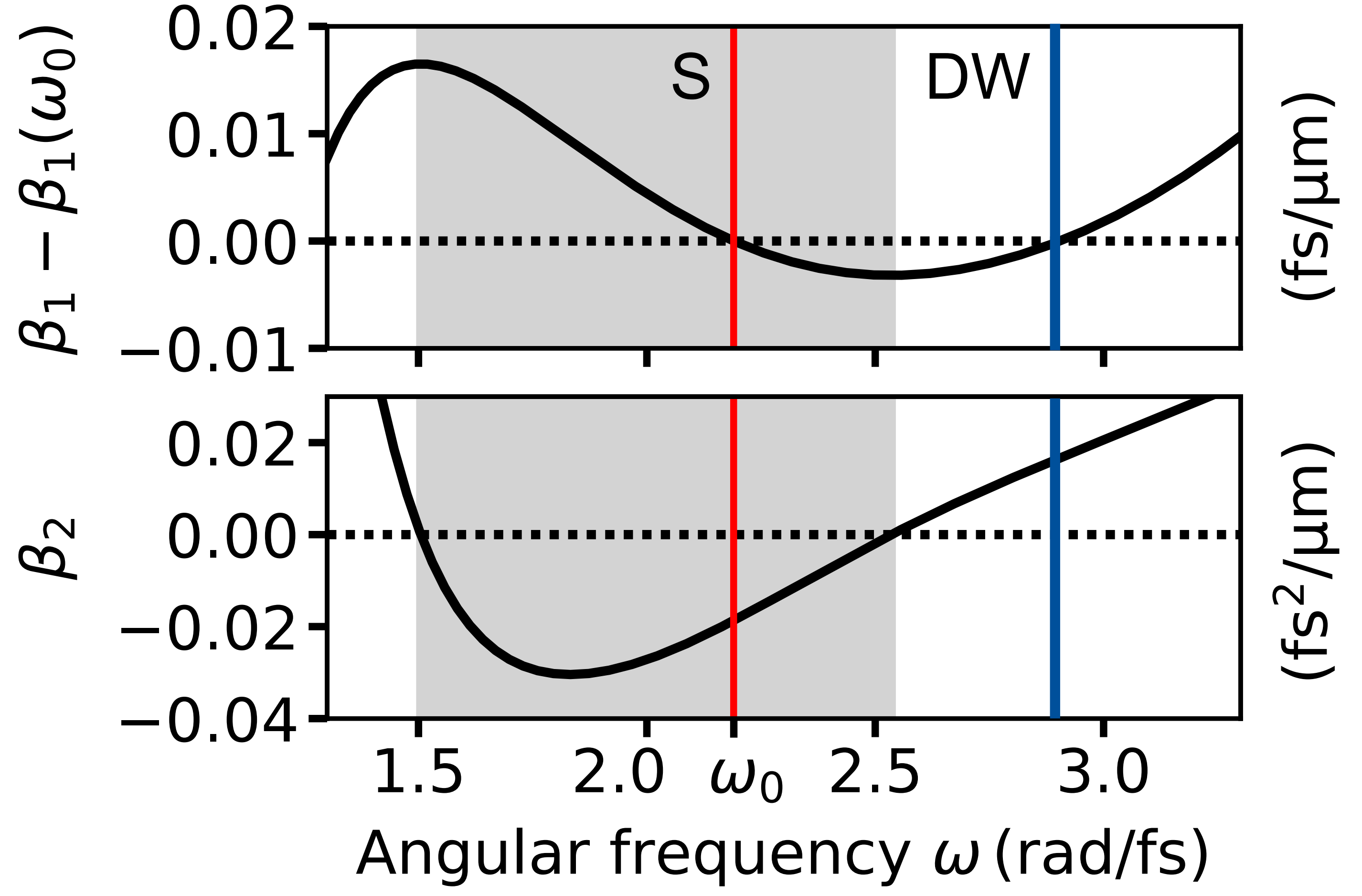




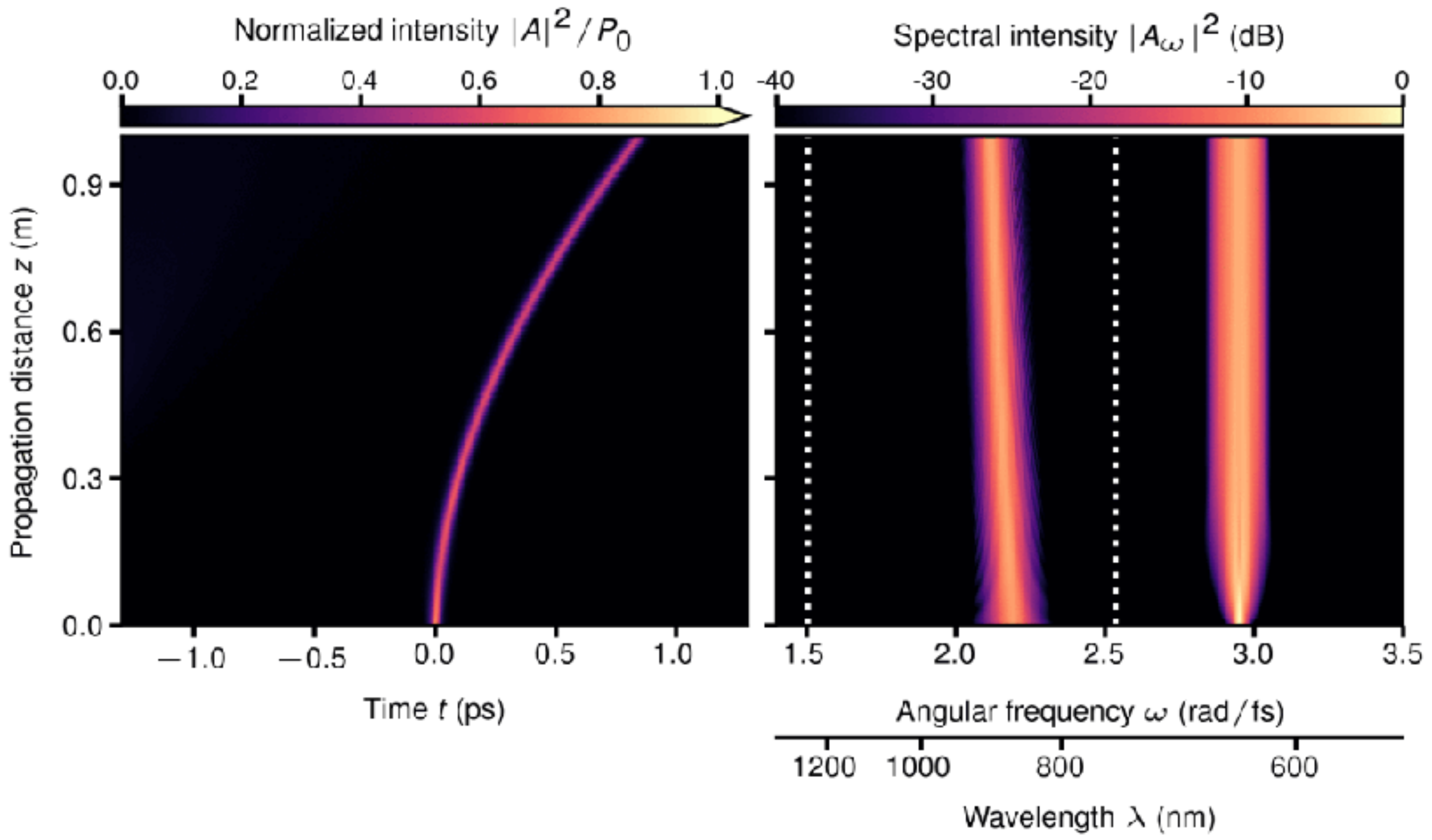
[Dudley, Genty, Coen; Rev. Mod. Phys. 78 (2006) 1135]

Switching concept enabled by nonlinear processes

- Customized to fit NL-PM-750 (NKT Photonics)
  [Melchert *et al.*; Commun. Phys. 3 (2020) 146]



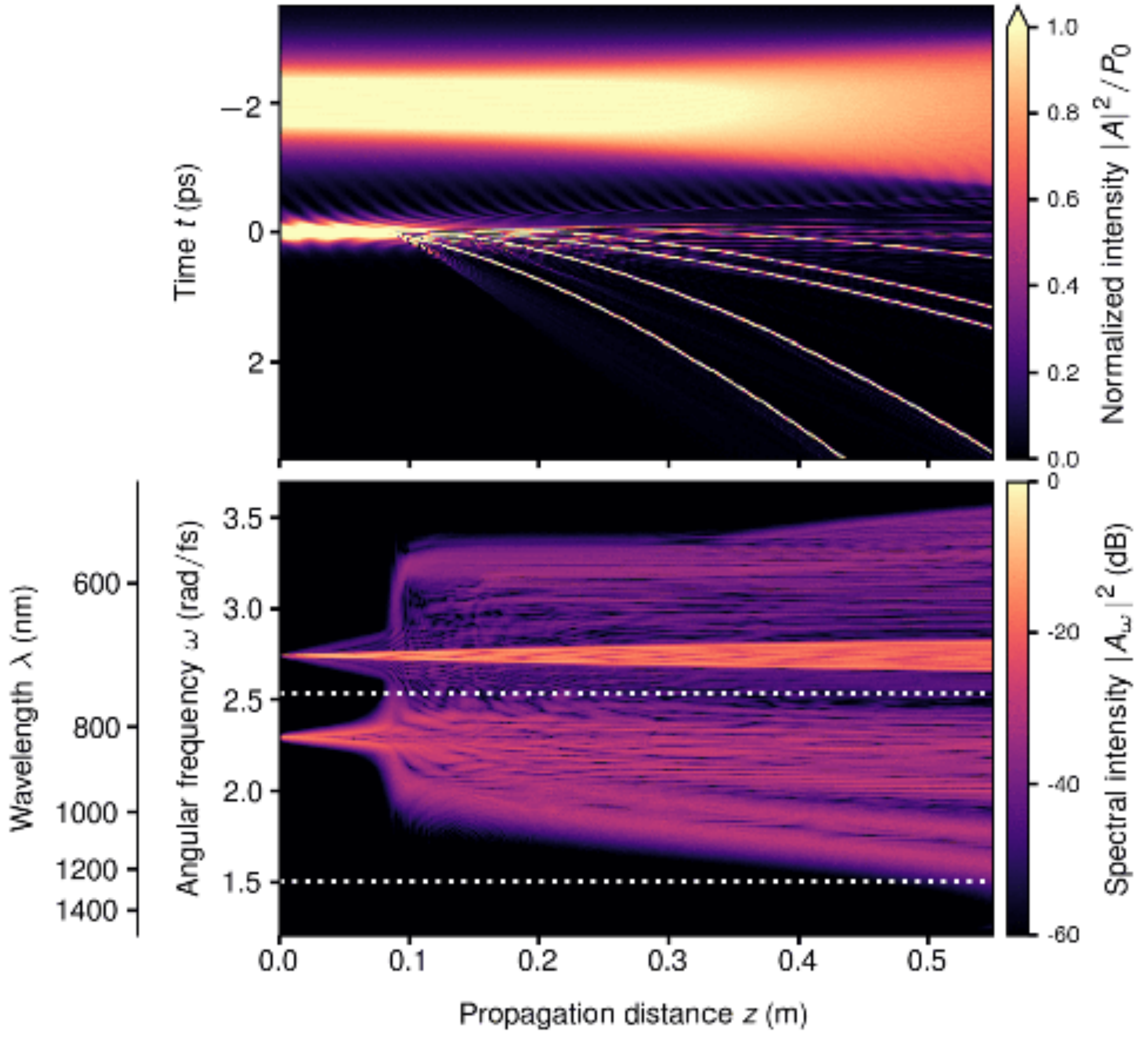
- Initial pulse delay affects self-frequency shift



Dispersive wave induced supercontinuum (SC) switching

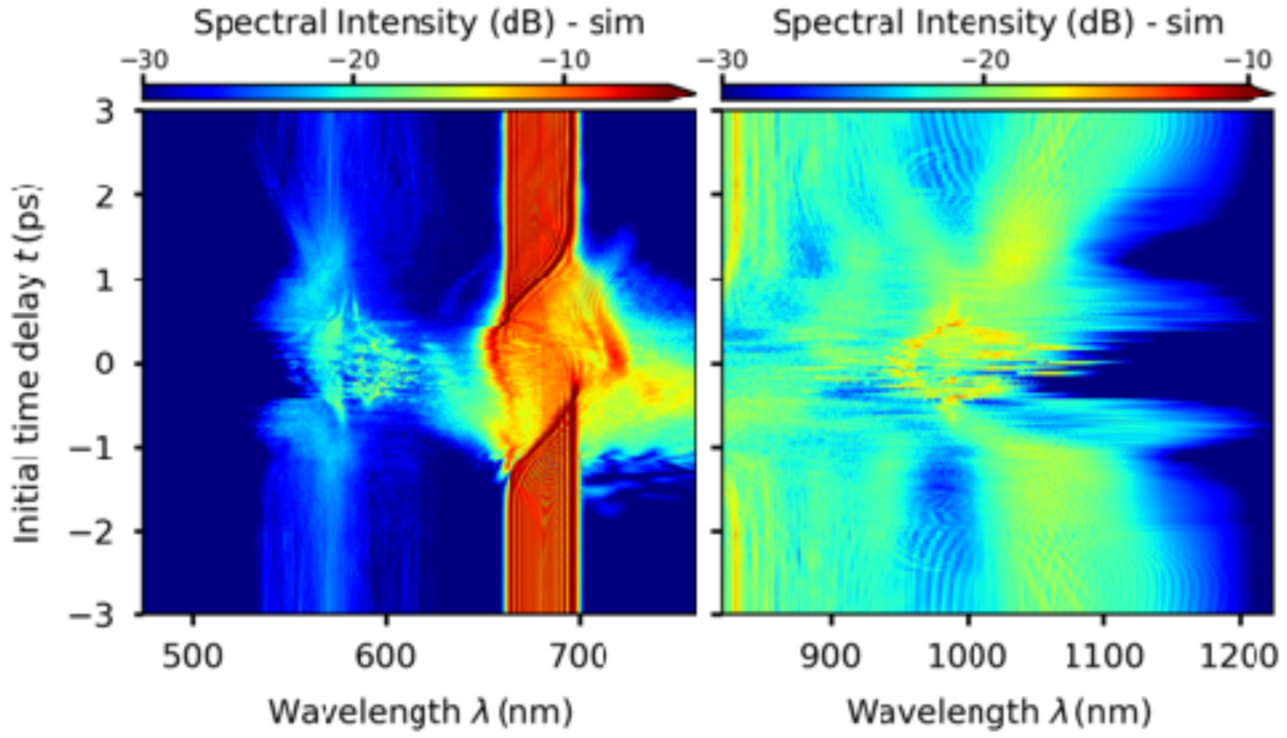
- Higher-order soliton + normally dispersive wave
  - Controlling different parts of soliton-fission induced SC spectra

(delay sweep for single instance)

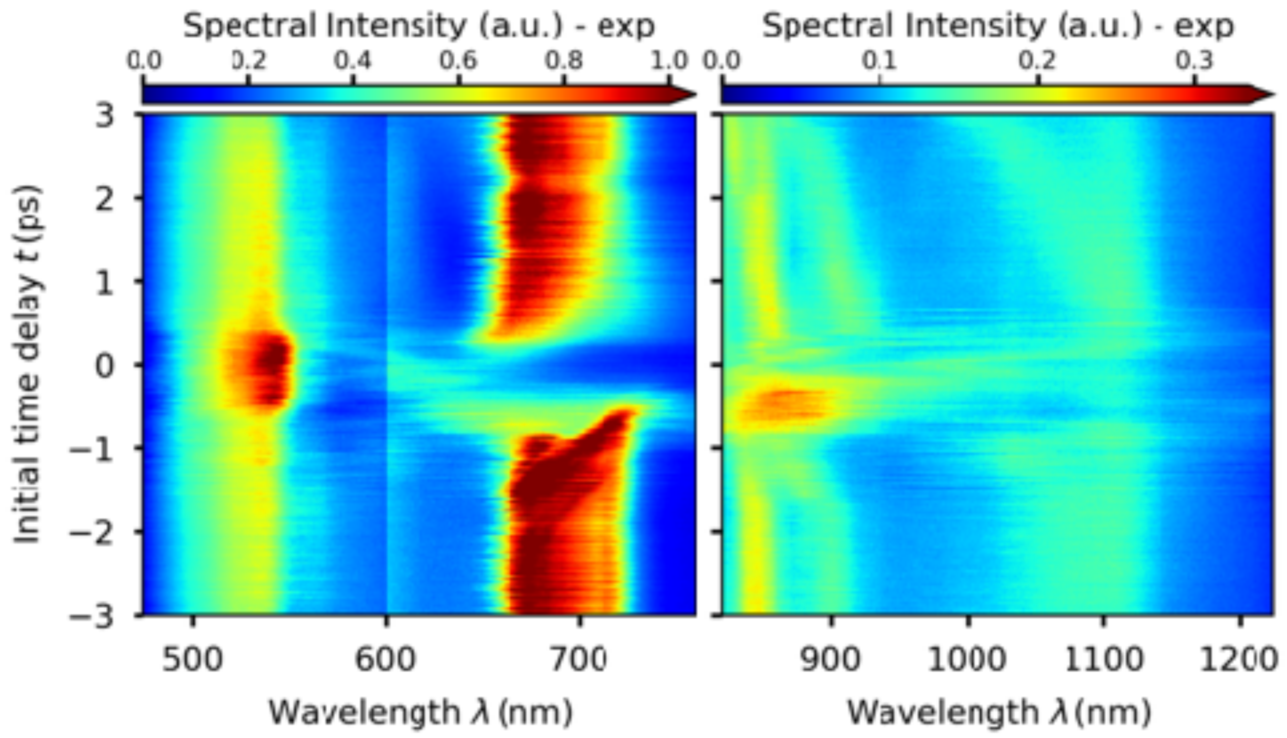


- Numerical simulations

(quantum noise averaged)



- Experiment



- All-optical switching logic

	Inputs		**Outputs (O_i)**		
Designation	DW	S	O_1	O_2	O_3
λ(nm)	680	800	540	680	1100
	0	0	0	0	0
	0	1	0	0	1
	1	0	0	1	0
	1	1	1	0	0
Functionality			S & DW	$\overline{S}$ & DW	S & $\overline{DW}$
Cascadability			-	✓	(✓)

S soliton, *DW* dispersive wave.

[Melchert *et al.*; Commun. Phys. 3 (2020) 146]

All-optical SC switching

- Exploits wave reflection mechanism
- Uses higher-order solitons
- Enables 3 AND-gate functionalities
- Femtosecond switching times

Part 3

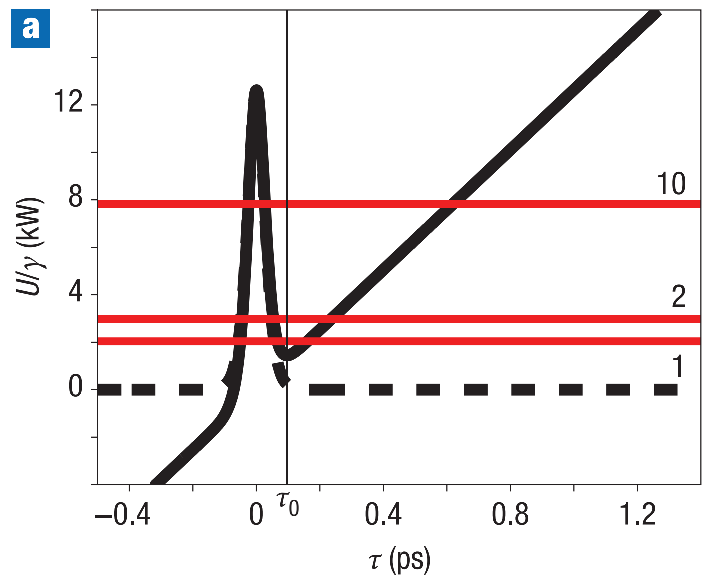
Two-frequency pulse compounds

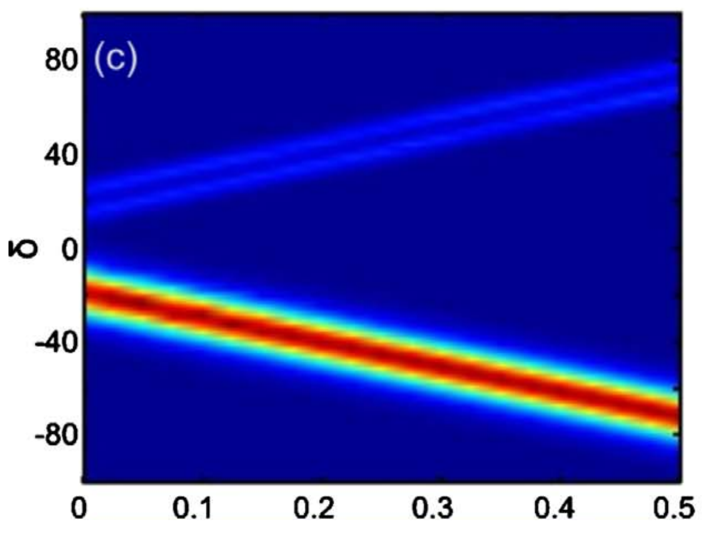
Trapping and soliton molecules with two frequencies

- Radiation trapping by *decelerating* soliton

  [Gorbach, Skryabin, Nature Photonics 1 (2007) 653]

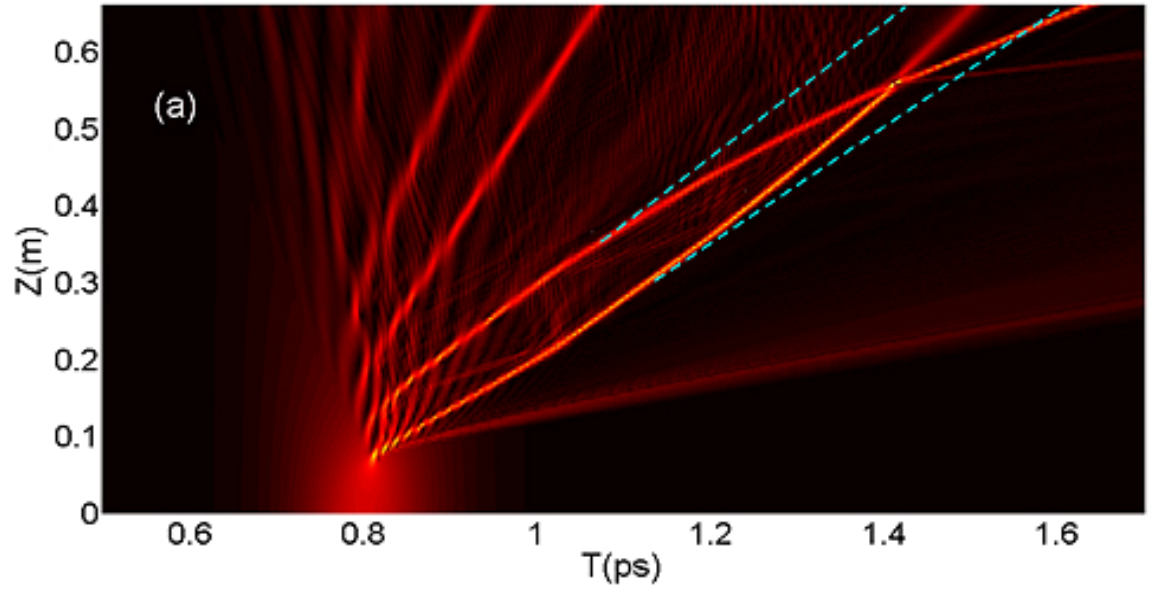
  [Gorbach, Skryabin, PRA 76 (2007) 053803]





- Trapping in *soliton-delimited cavities*

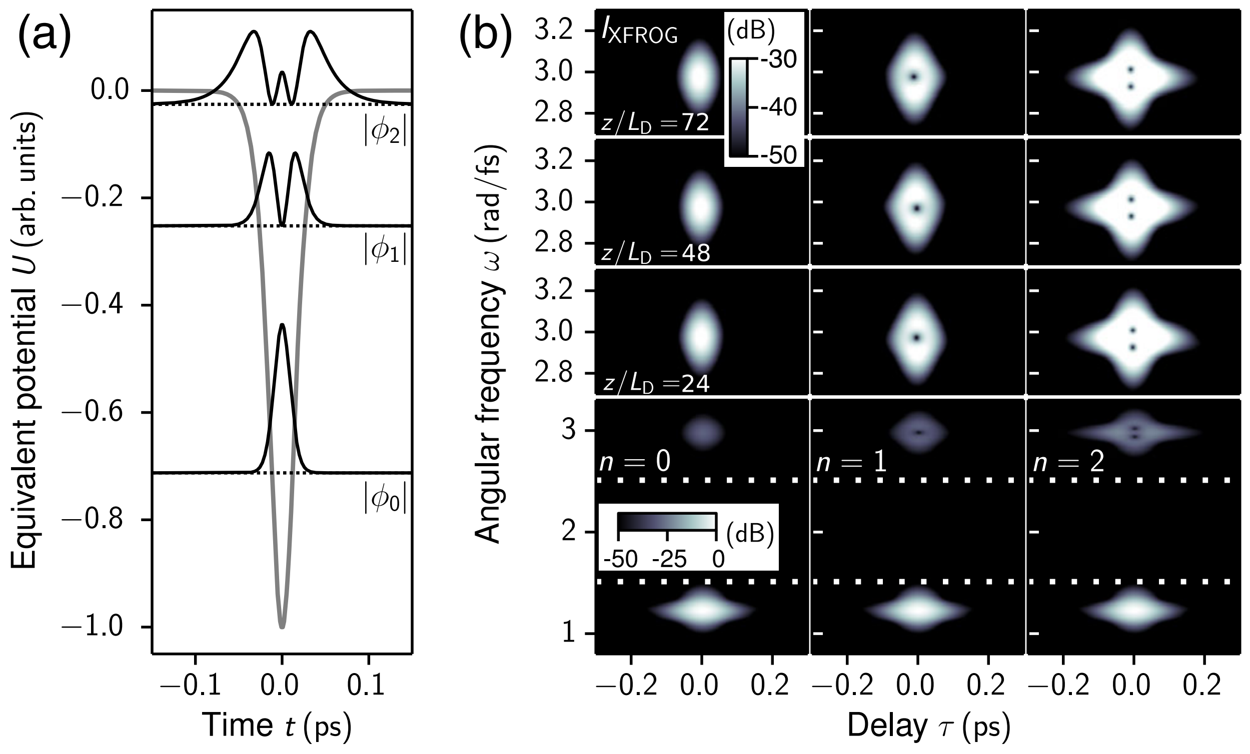
  [Driben, Yulin, Efimov, Malomed, Optics Express 21 (2013) 19091]



  [Wang, Mussot, Conforti, Zeng, Kudlinski, Opt. Lett. 40 (2015) 3320]

- Soliton molecules with two frequencies

  [Melchert *et al.*, PRL 123 (2019) 243905]

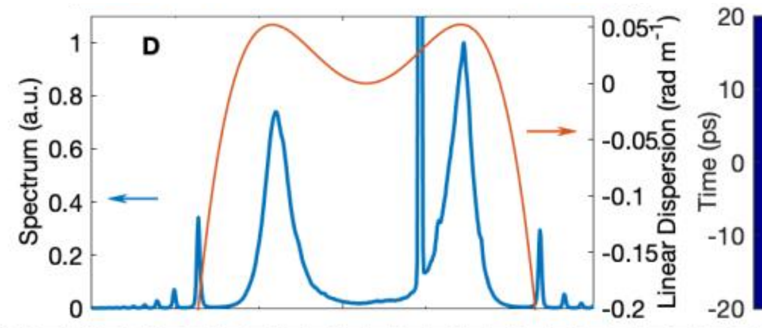


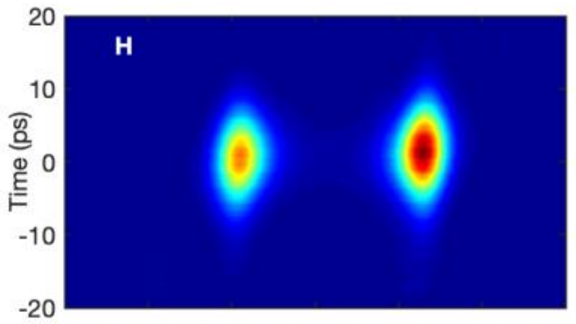
generalized dispersion Kerr solitons

[Tam, Alexander, Hudson, Blanco-Redondo, de Sterke, PRA 101 (2020) 043822]

recent experimental demonstration:

[Lourdesamy, Runge, Alexander, Hudson,
Blanco-Redondo, de Sterke, arxiv:2007.01351]





Details of the considered propagation constant

$$i\partial_z \mathcal{E}_\omega + \beta(\omega)\mathcal{E}_\omega + \frac{3\omega^2\chi}{8c^2\beta(\omega)}\left(|\mathcal{E}|^2\mathcal{E}\right)_{\omega>0} = 0$$

- Propagation constant

$$\beta(\omega) = \frac{\omega}{c}\text{Re}\left[n(\omega)\right]$$

$$\beta_1(\omega) = \partial_\omega \beta(\omega)$$ (group delay)

$$\beta_2(\omega) = \partial_\omega^2 \beta(\omega)$$ (group velocity dispersion; GVD)

$$\beta_3(\omega) = \partial_\omega^3 \beta(\omega)$$

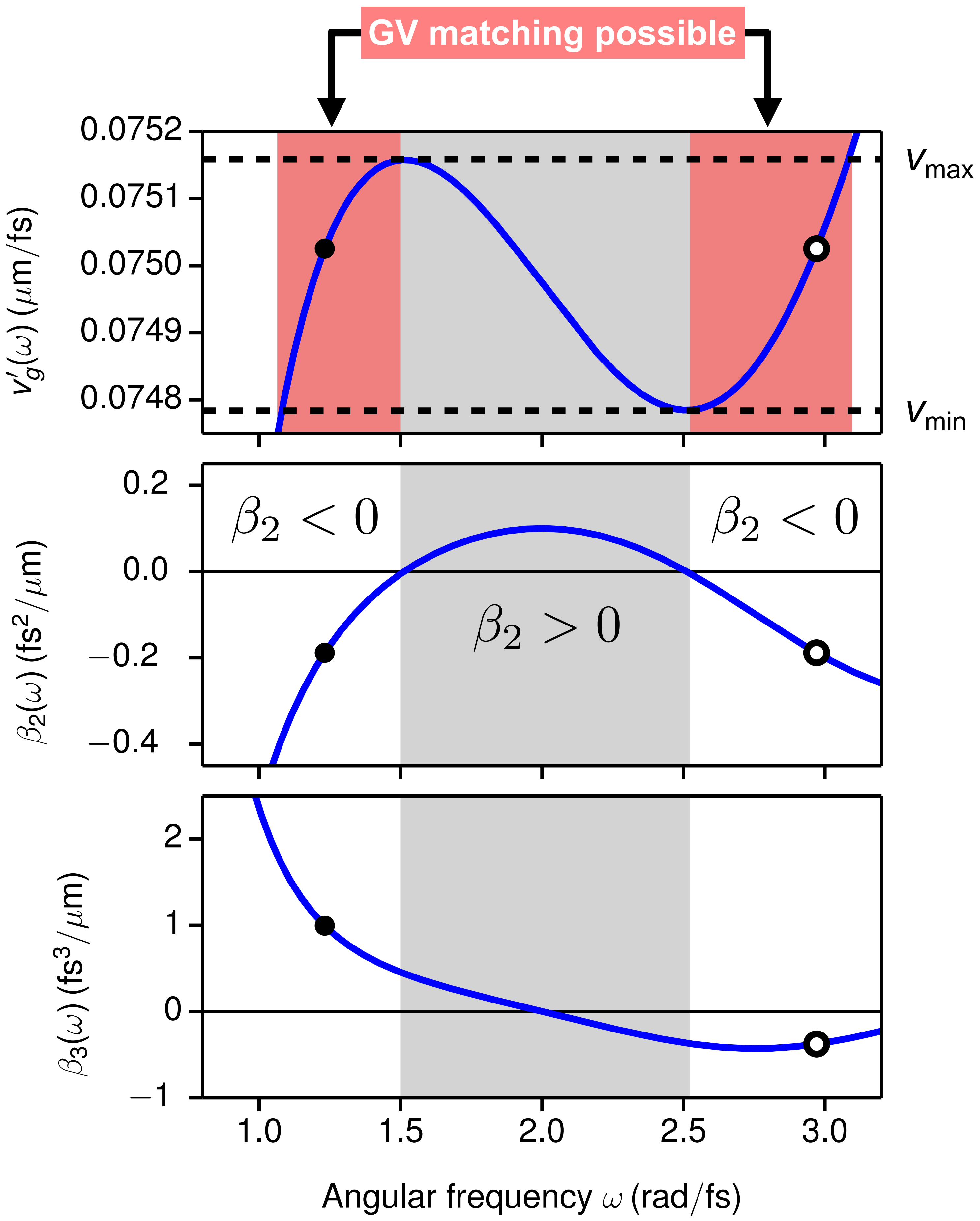
$$v_g(\omega) = 1/\beta_1(\omega)$$ (group velocity; GV)

$$v_g'(\omega) = \left[\beta_1(\omega) - \frac{\beta_2(\omega)}{\omega t_0^2} + \frac{\beta_3(\omega)}{6t_0^2}\right]^{-1}$$

[Haus, Ippen, Opt. Lett. 26 (2001) 1654]
[Pickartz, Bandelow, Amiranashvili, PRA 94 (2016) 033811]

- Zero-dispersion frequencies

$$(\omega_{Z1}, \omega_{Z2}, \omega_{Z3}) = (1.511, 2.511, 5.461)\,\text{rad/fs}$$



Weak trapped states in solitary-wave well

- Linearised eigenvalue problem

$$\left[-\frac{|\beta_2'|}{2}\frac{d^2}{dt^2}+V(t)\right]\phi_n(t)=\kappa_n\phi_n(t)$$

(primes indicate quantities calculated at ω_{GVM2})

[Melchert *et al.*, PRL 123 (2019) 243905]

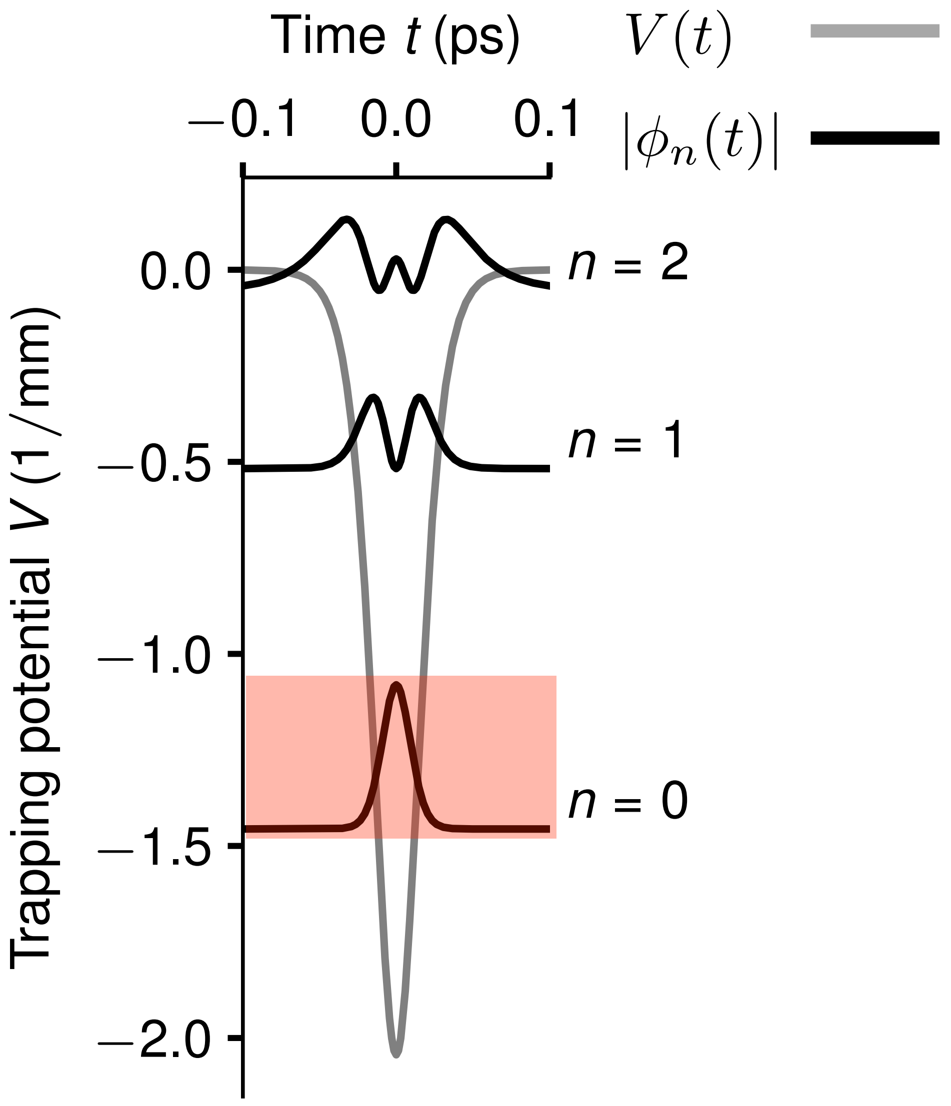
trapping potential:

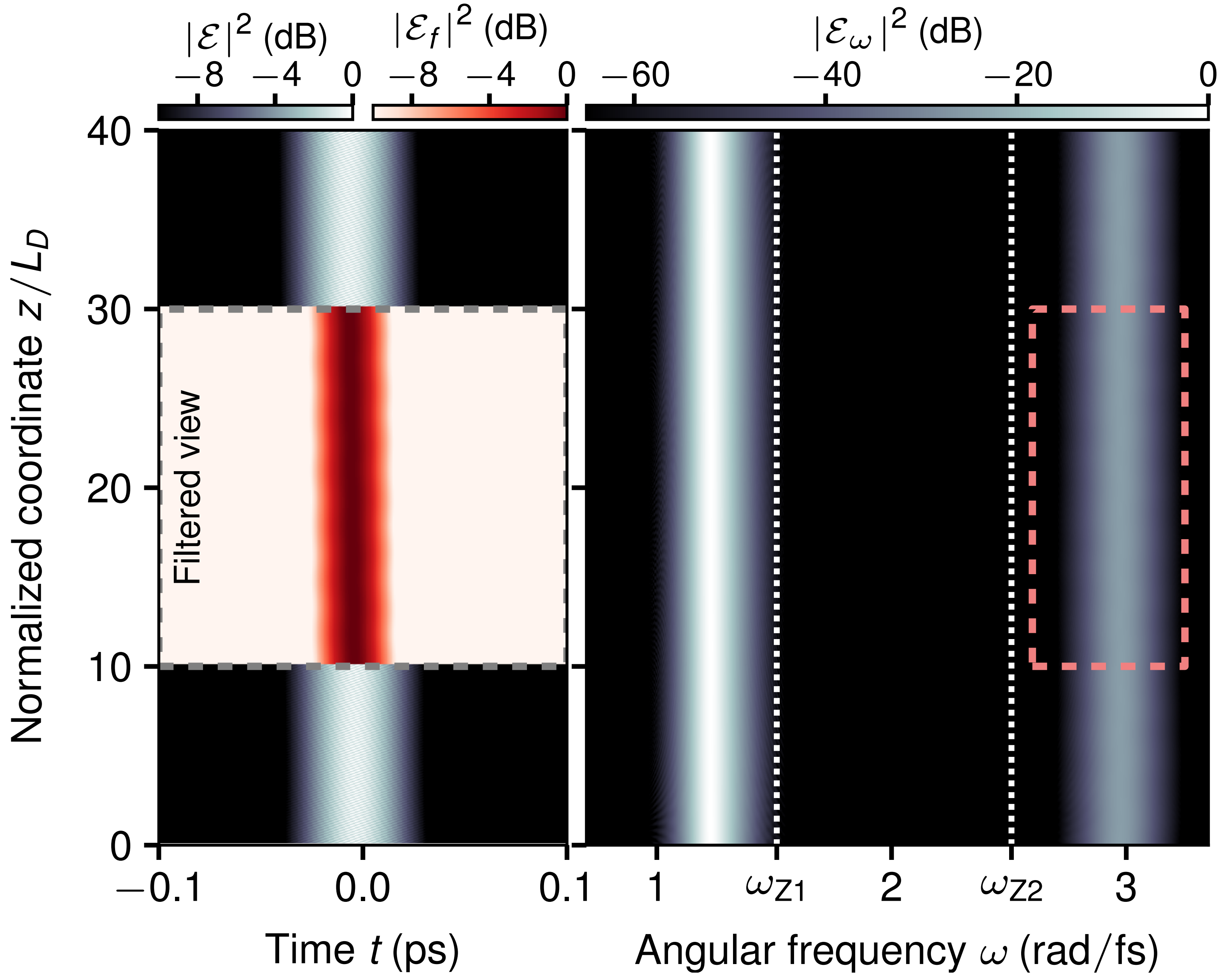
$$V(t)=-\frac{|\beta_2'|}{2}\frac{\nu(\nu+1)}{t_{\mathrm{S}}^2}\mathrm{sech}^2\left(t/t_{\mathrm{S}}\right)$$

- similar to sech² potential well in 1D quantum scattering

[Landau, Lifshitz, *Quantum Mechanics* (1981)]

[Lekner, Am. J. Phys. 75 (2007) 1151]





filtered view:

$$\mathcal{E}_f(z,t)=\sum_{\omega\in\square}\mathcal{E}_\omega(z)e^{-i\omega t}$$

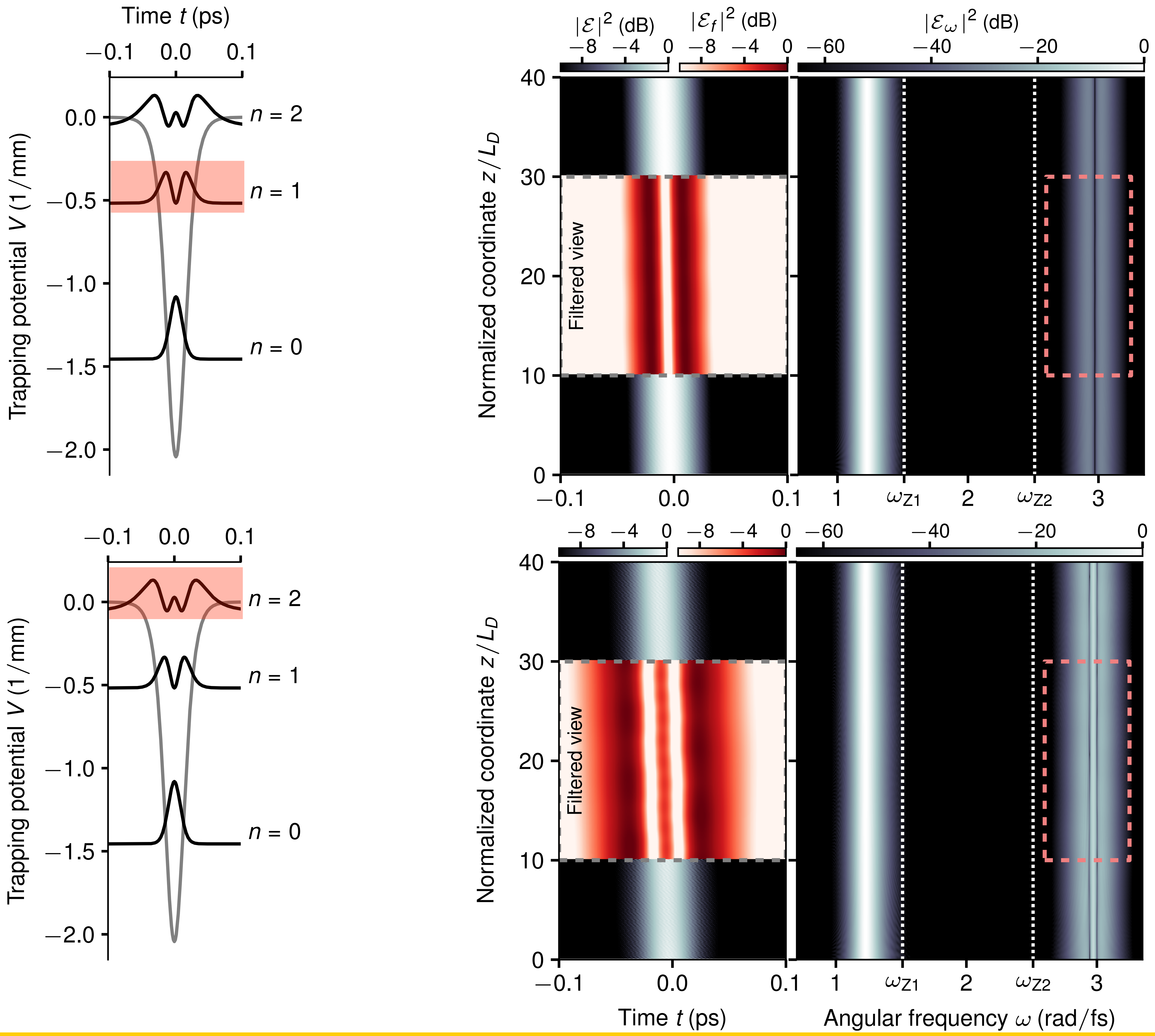
[Melchert *et al.*, PRL 123 (2019) 243905]

Weak trapped states in solitary-wave well

- Trapped state of order n = 1:

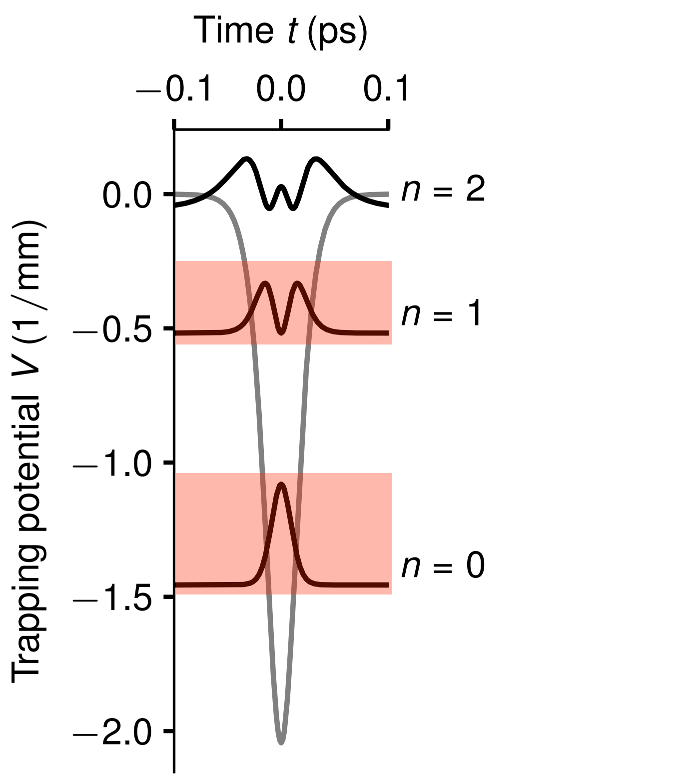
- Trapped state of order n = 2:

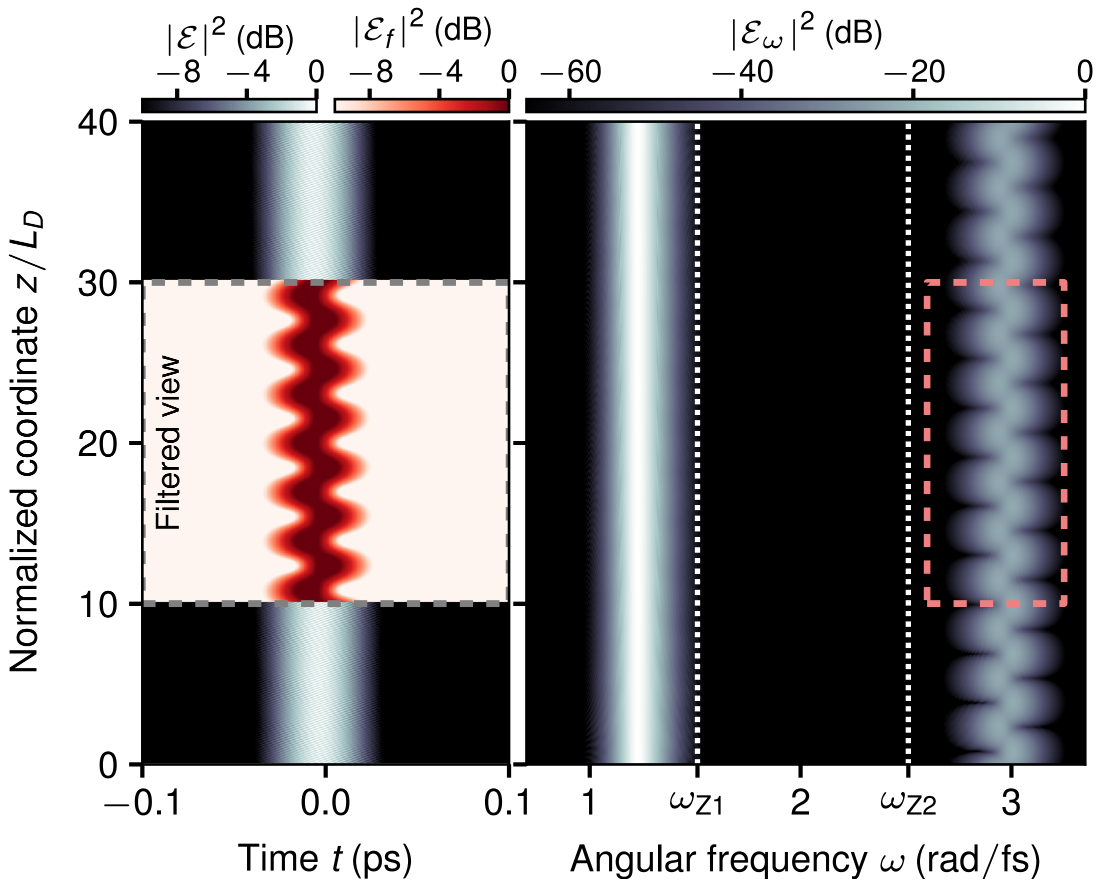
➡ Stable propagation over many soliton periods



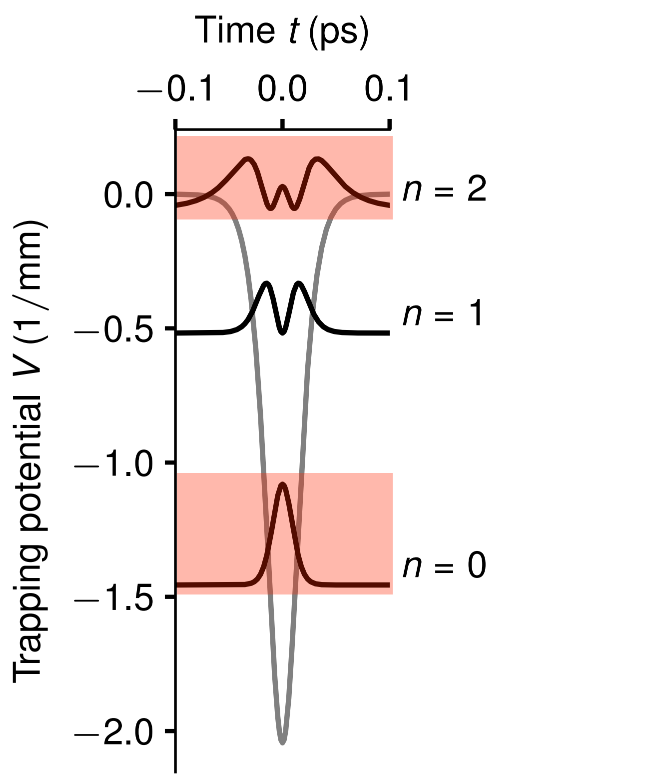
PhoeniXD

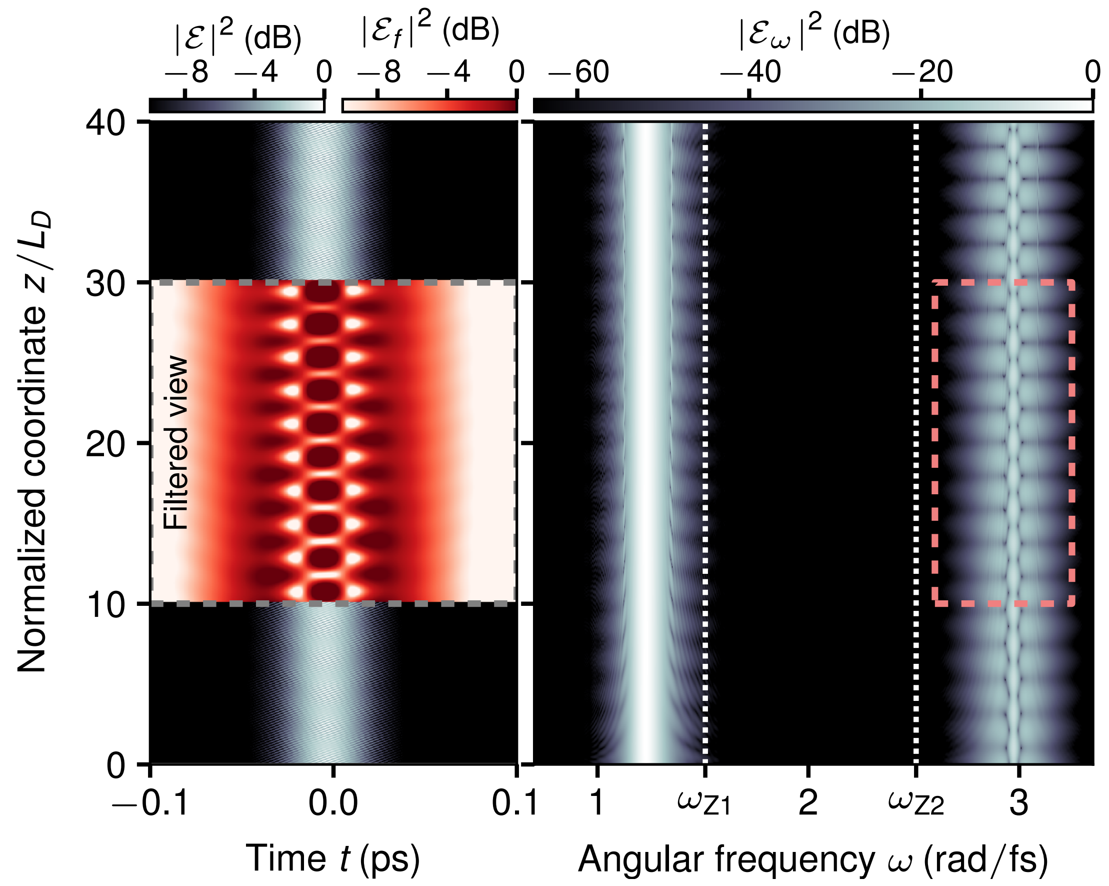
Simultaneous propagation of *multiple* trapped states - coherent dynamics



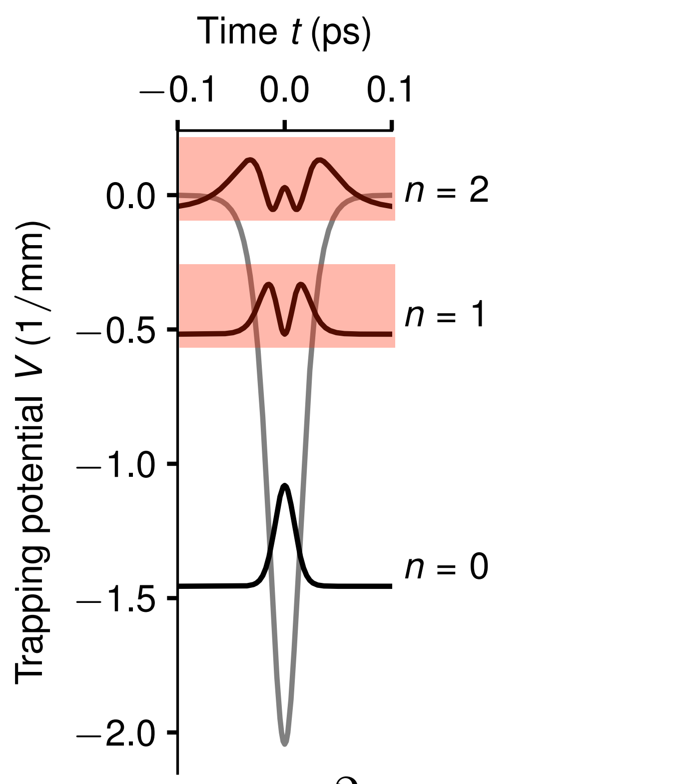


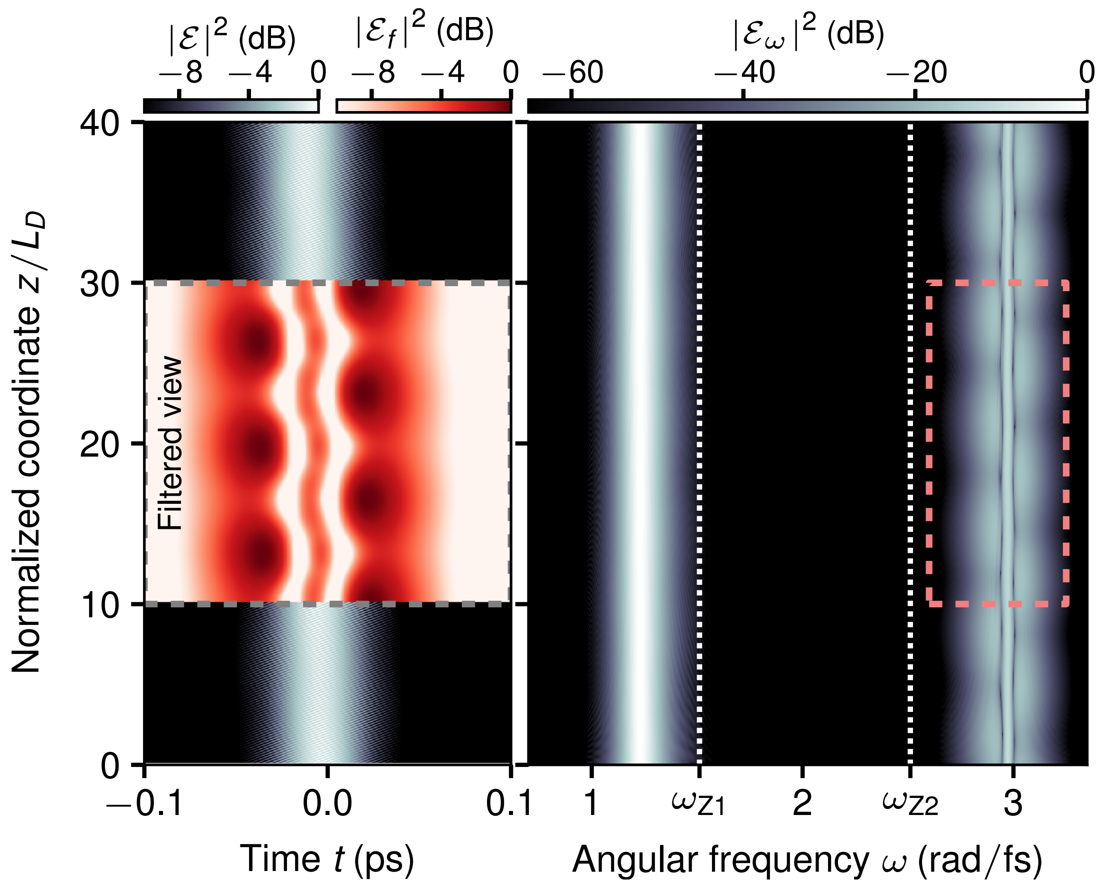
$$\frac{z}{L_D} = \frac{2\pi}{|\kappa_0 - \kappa_1| L_D} \approx 3.16$$



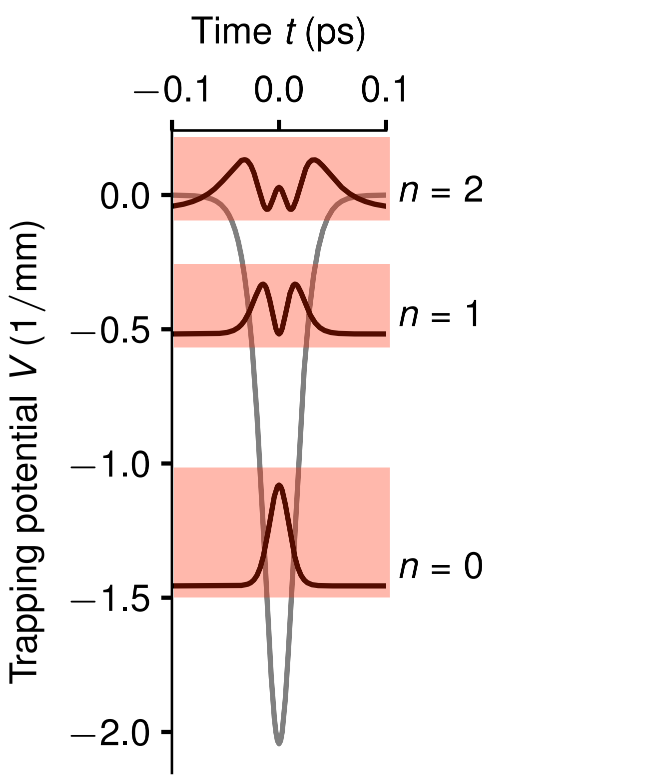


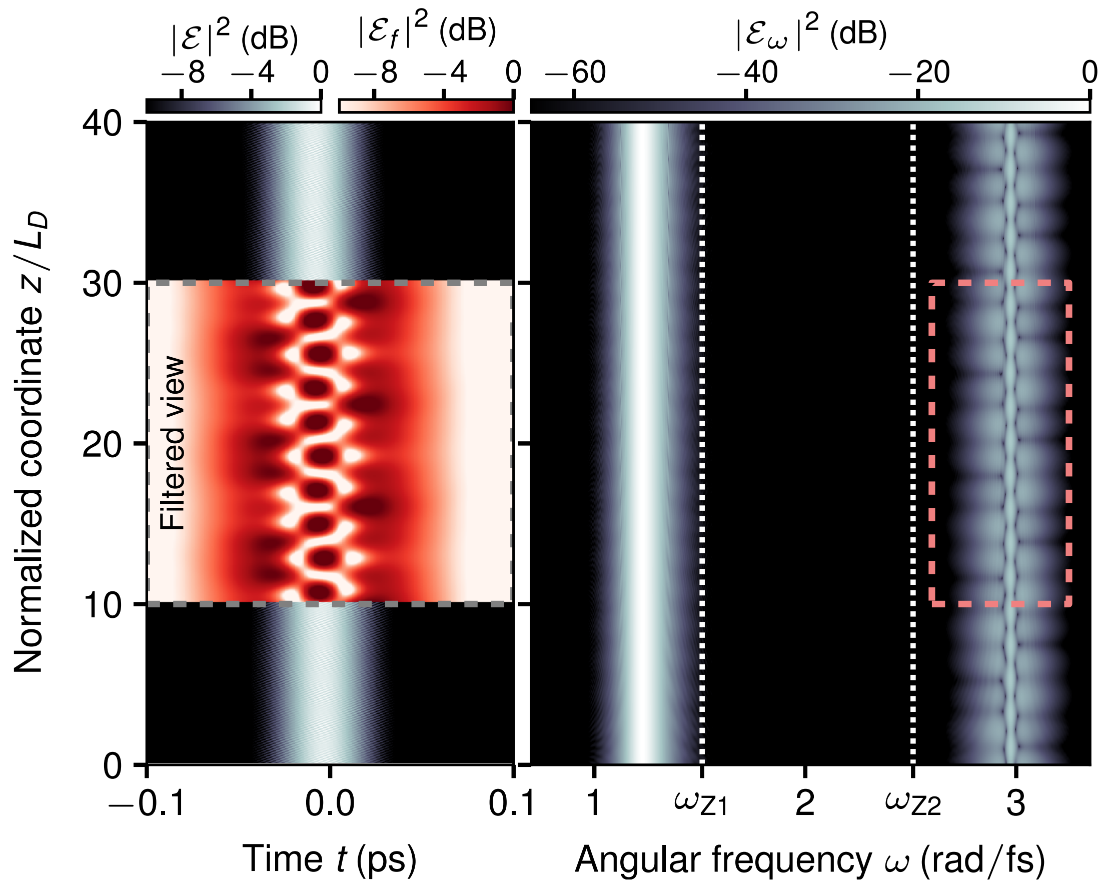
$$\frac{z}{L_D} = \frac{2\pi}{|\kappa_0 - \kappa_2| L_D} \approx 2.11$$





$$\frac{z}{L_D} = \frac{2\pi}{|\kappa_1 - \kappa_2| L_D} \approx 6.37$$

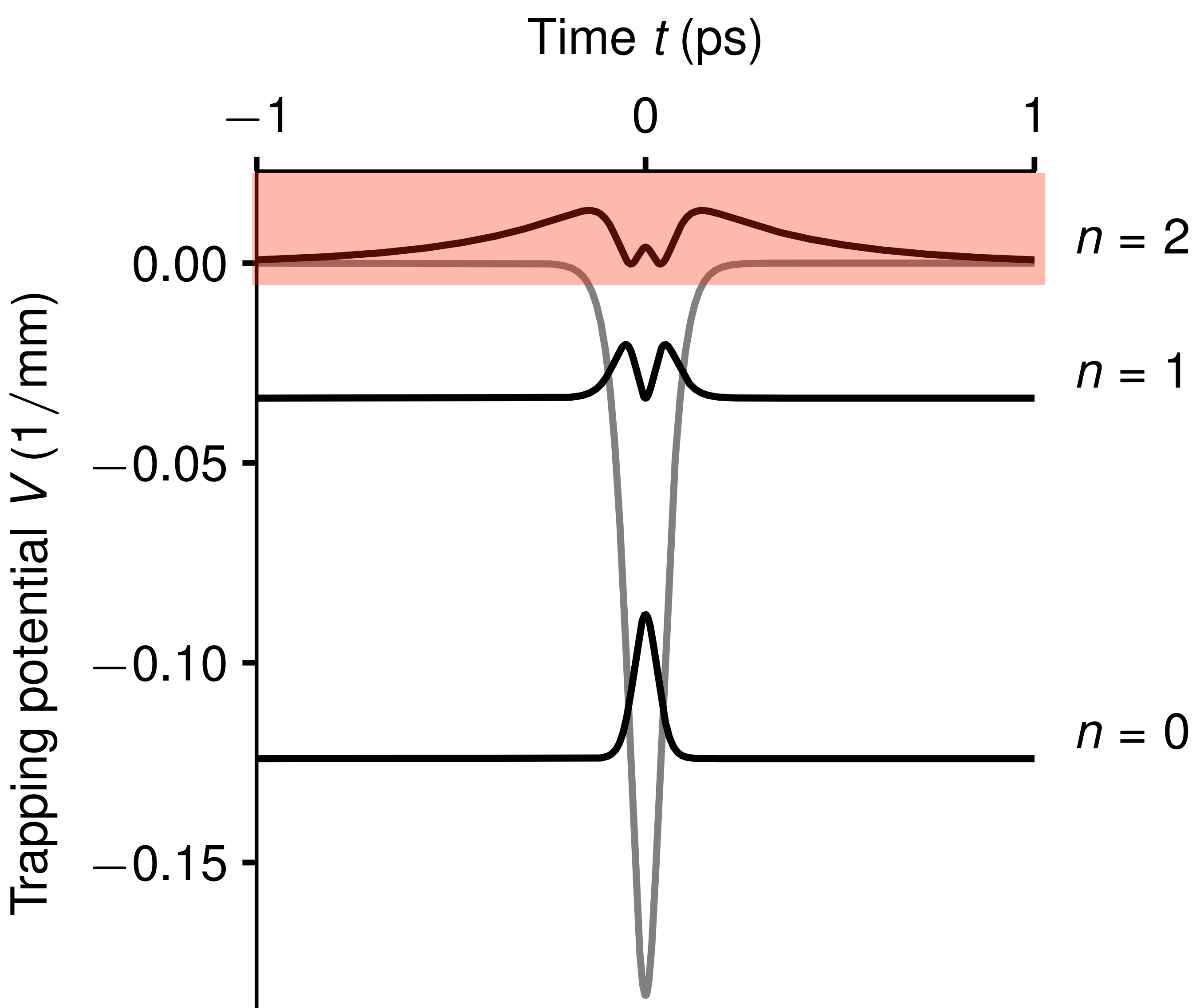


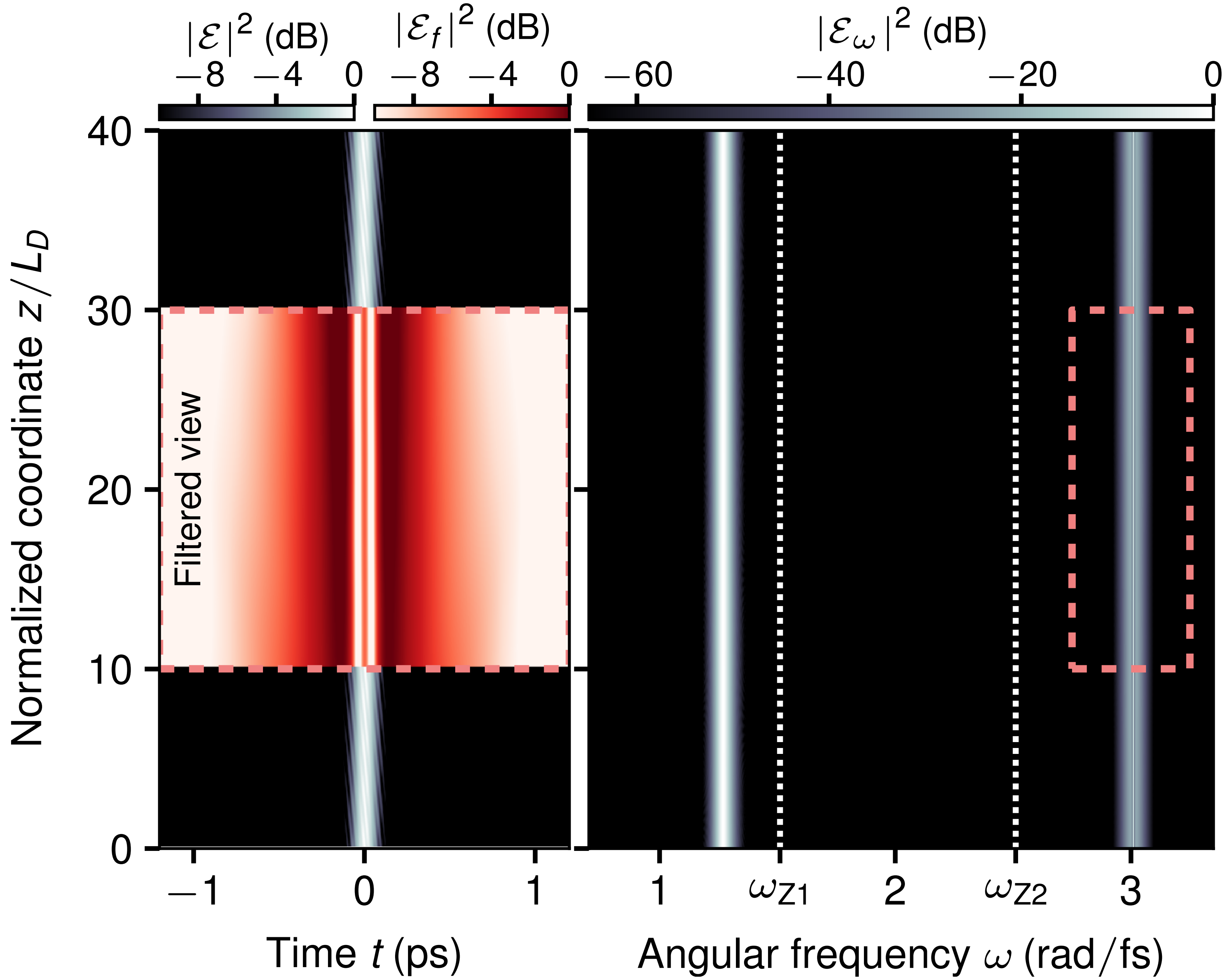


PhoenixD

Extreme states of light — optical halos

- Example for $\omega_\mathrm{S} = 1.27\,(\mathrm{rad/fs})\quad t_\mathrm{S} = 60\,\mathrm{fs}$





- Root-mean-square duration

$$t_\mathrm{rms}^\mathrm{halo} \approx 8 \times t_\mathrm{rms}^\mathrm{S}$$

PhoenixD

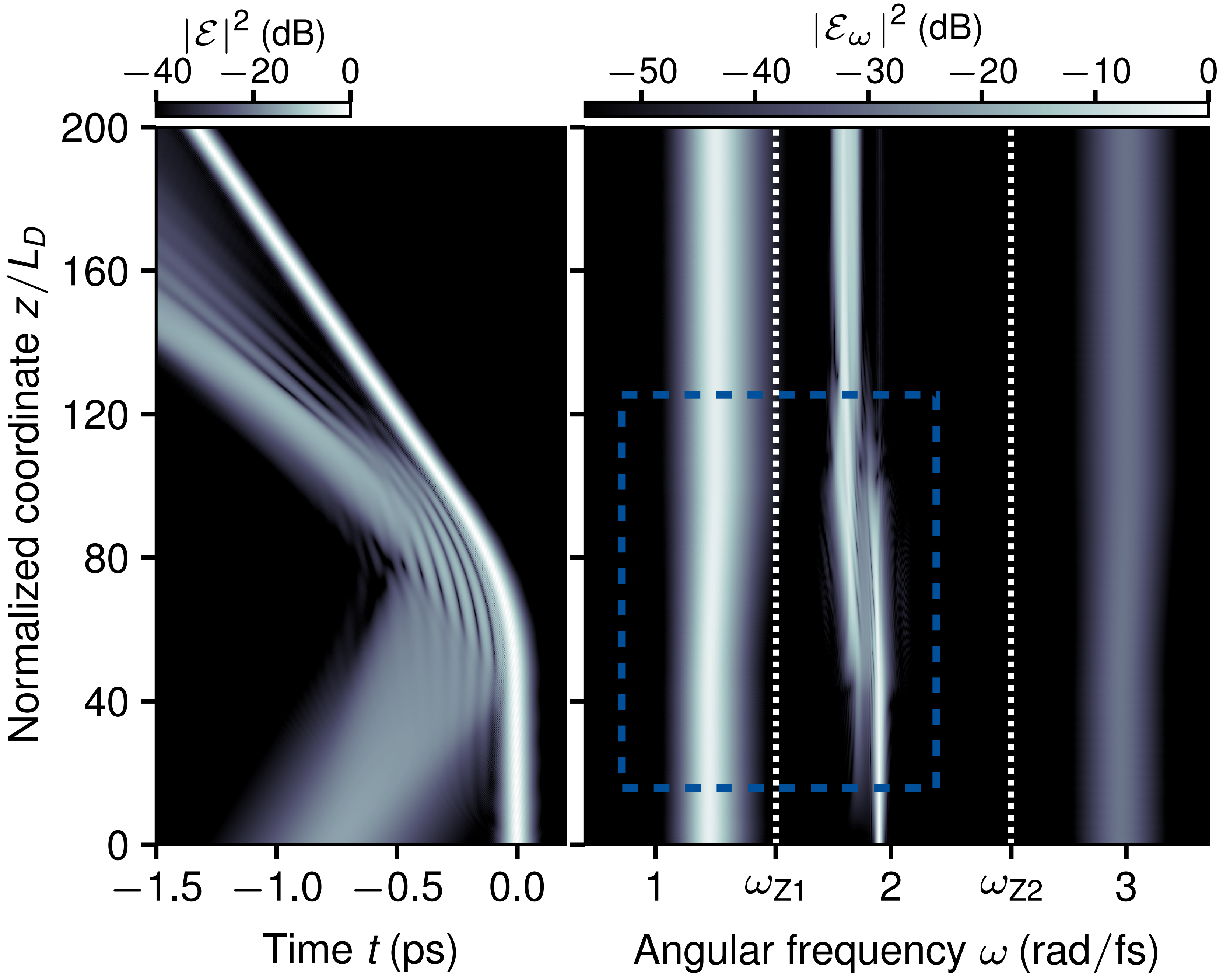
Robustness against perturbation

- interaction with normally dispersive wave

- general wave-reflection mechanism
  - co-propagation with similar GV
  - strong repulsive interaction

  [Demircan *et al.*, PRL 106 (2011) 163901]
  [Smith, Math. Proc. Camb. Phil. Soc 78 (1975) 517]
  [de Sterke, Opt. Lett. 17 (1992) 914]
  [Philbin *et al.*, Science 319 (2008) 1367]
  [Faccio, Cont. Phys. 1 (2012) 1]

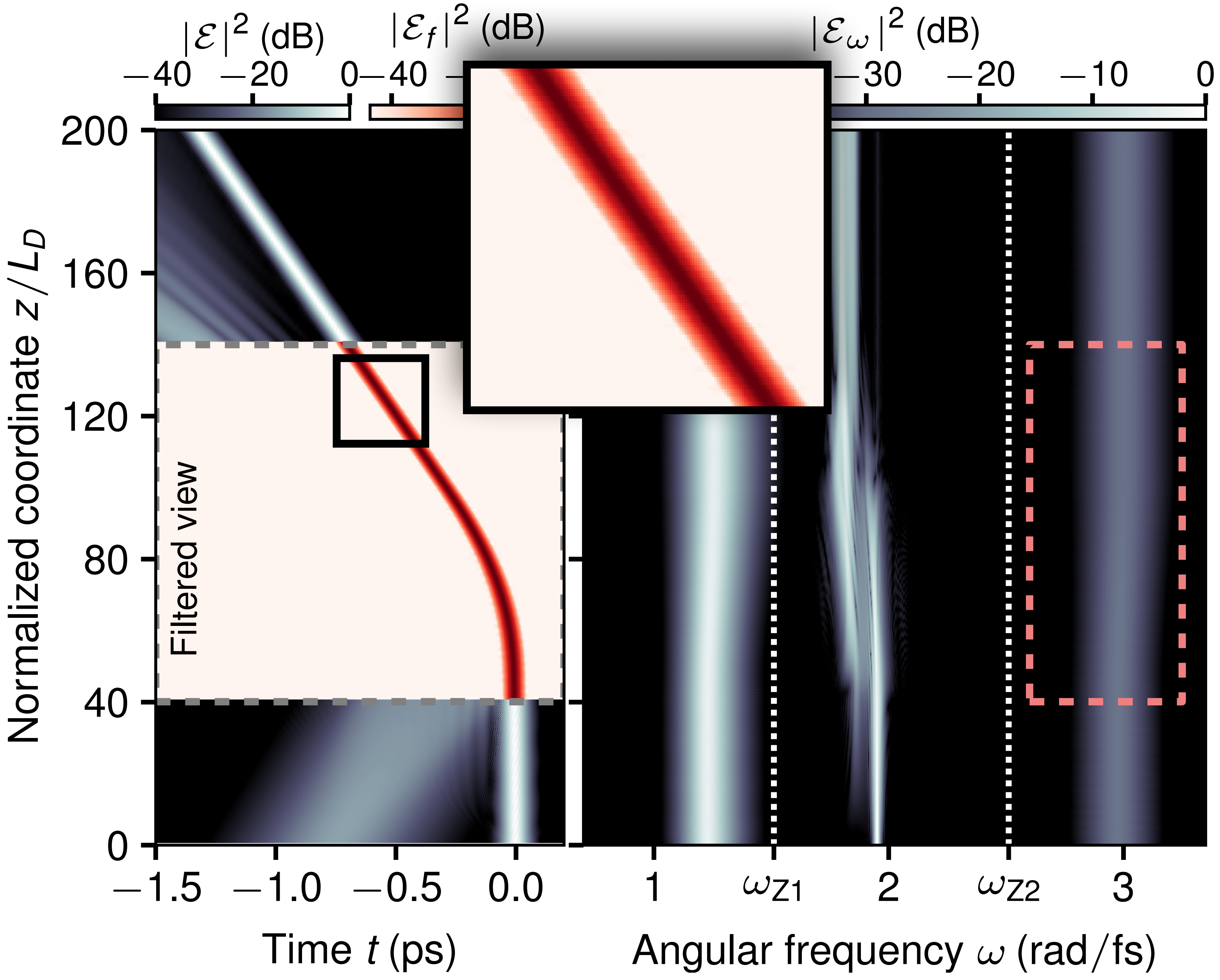
  - shift of soliton center frequency



PhoenixD

Robustness against perturbation

- interaction with normally dispersive wave
- observation
  - trapping potential experiences acceleration
  - trapped states are dragged along
  - frequency up-shift; no radiation

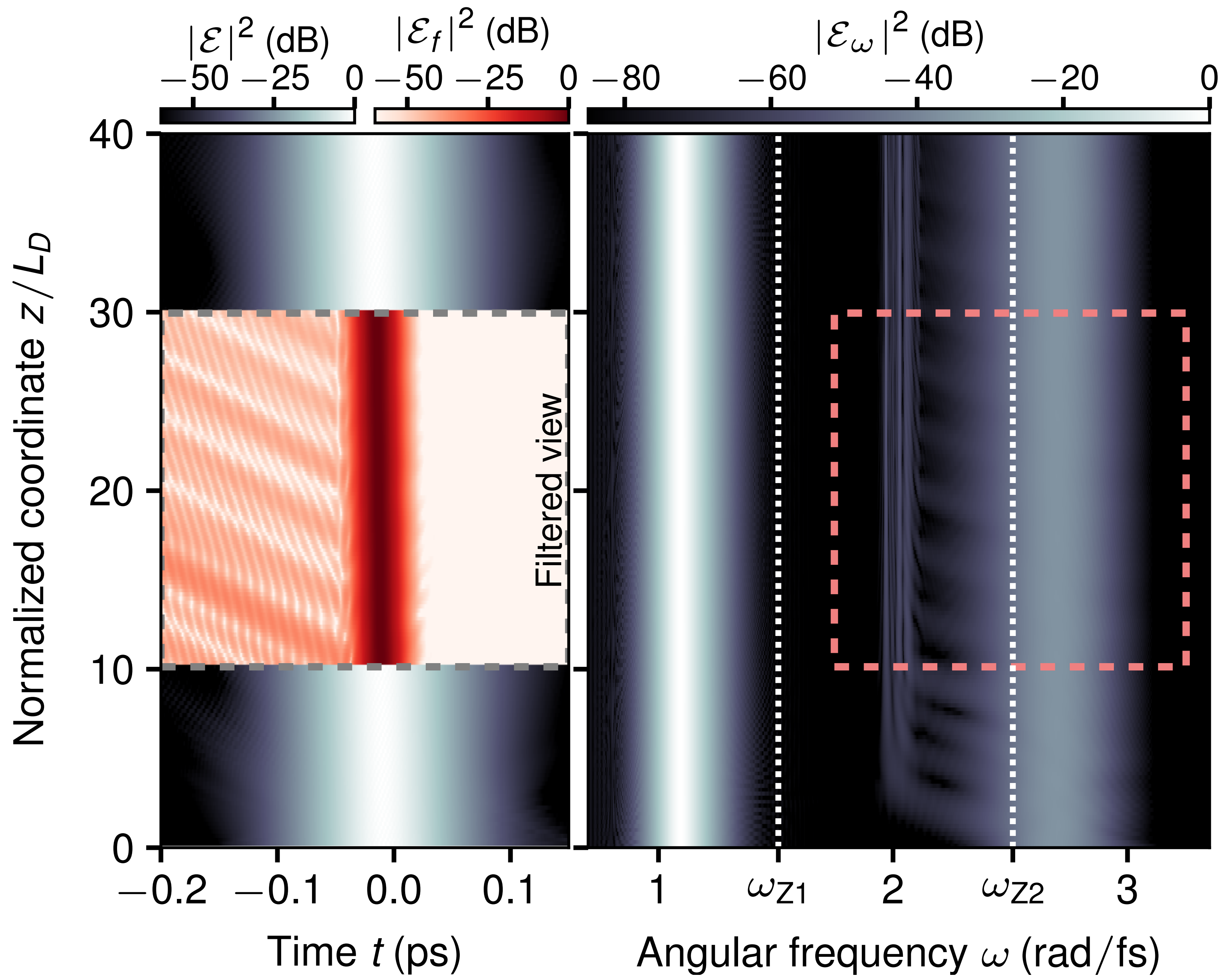


- Raman induced frequency shift of soliton
  - trapped states persist

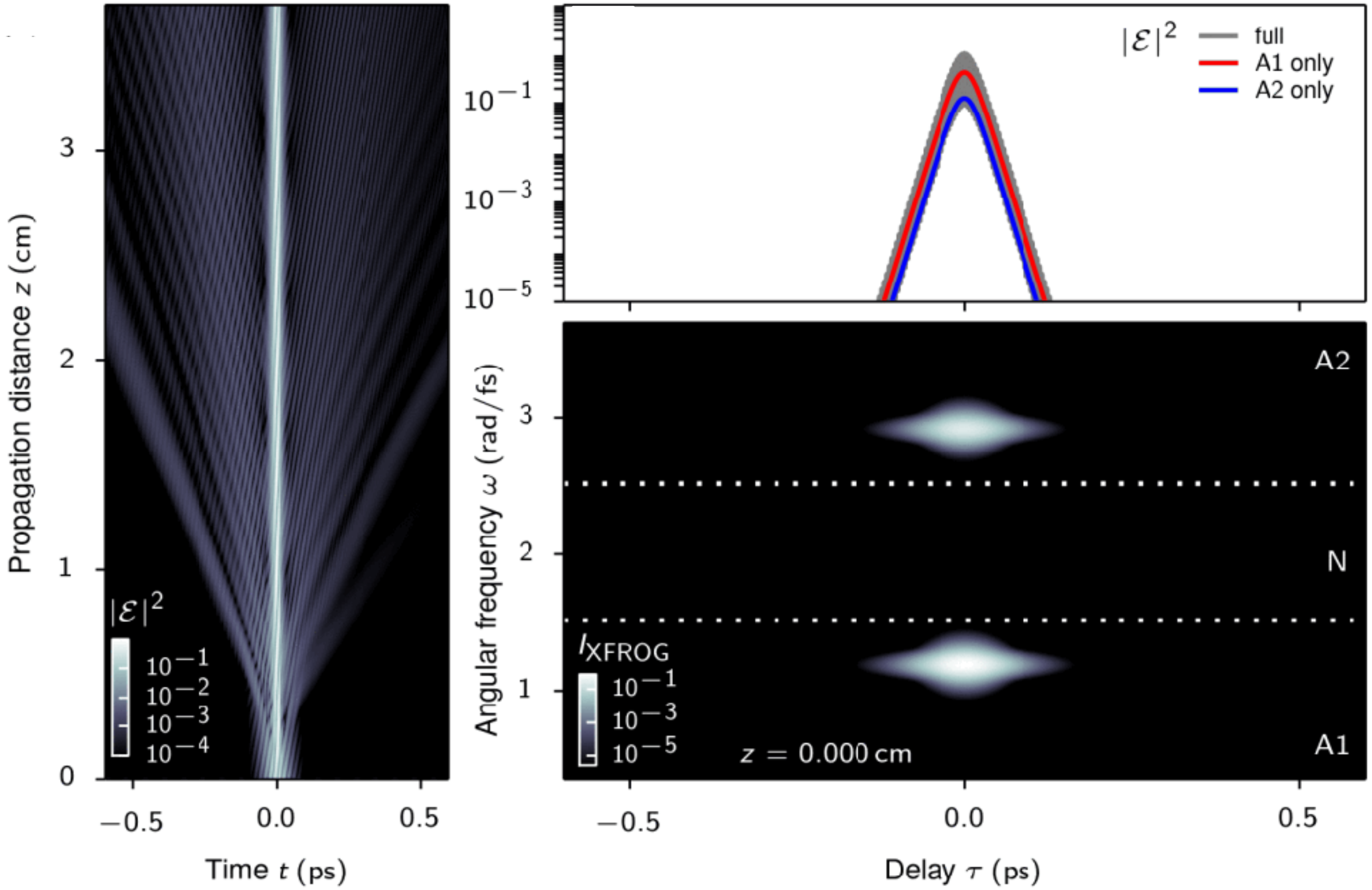
[Willms *et al.*, in preparation]

Leaking trapped states

- Observation close to zero-dispersion points
  - trapped state remains localised
  - trapped state emits dispersive waves
  - *leak-effect* occurs close to zero dispersion point

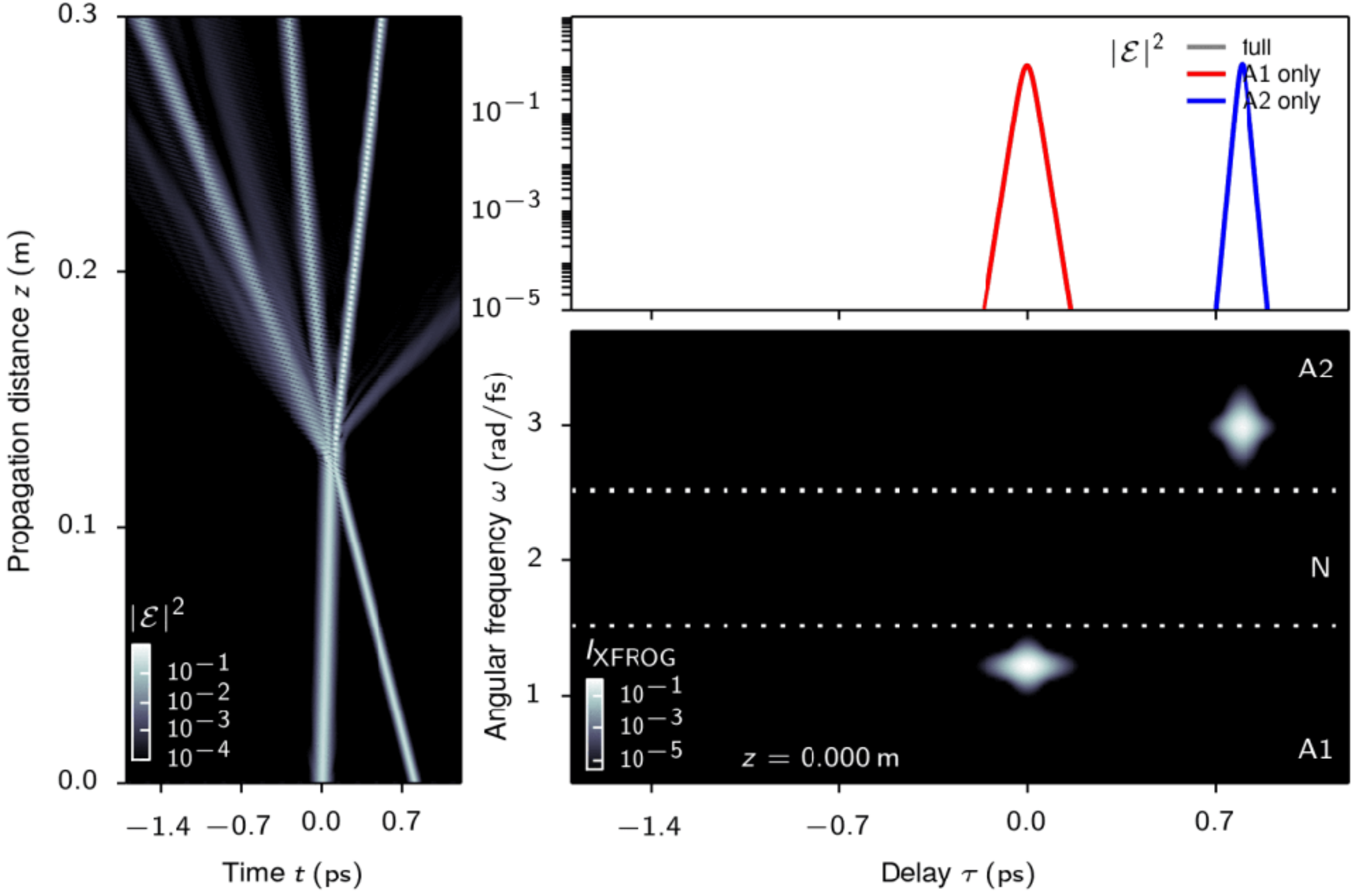


Generating molecule states by direct superposition of solitons



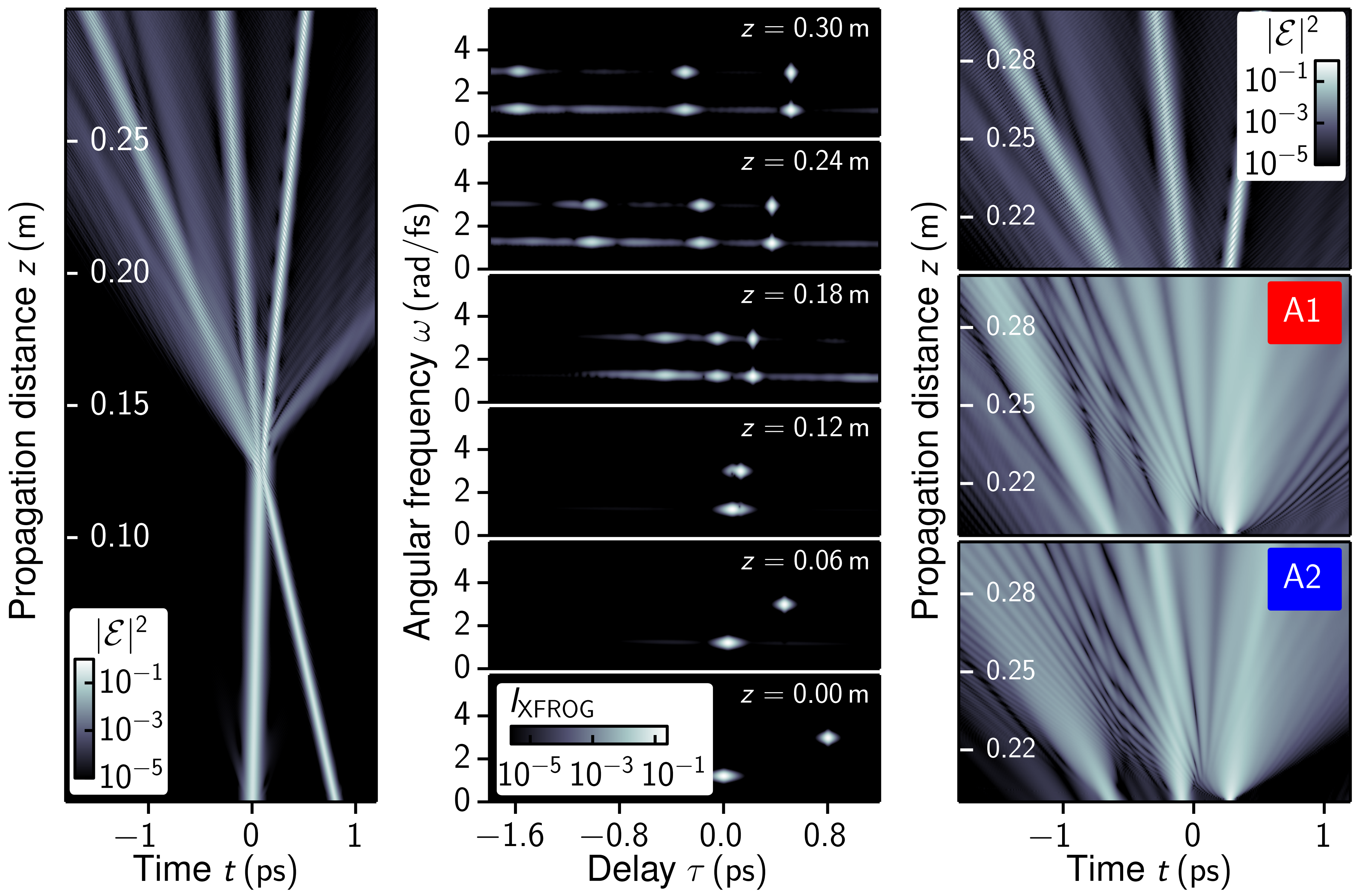
[Melchert *et al.*, PRL 123 (2019) 243905]

Generating molecule states through soliton-soliton collisions



[Melchert *et al.*, PRL 123 (2019) 243905]

Molecule states exhibit binding force



mutual interaction enabled

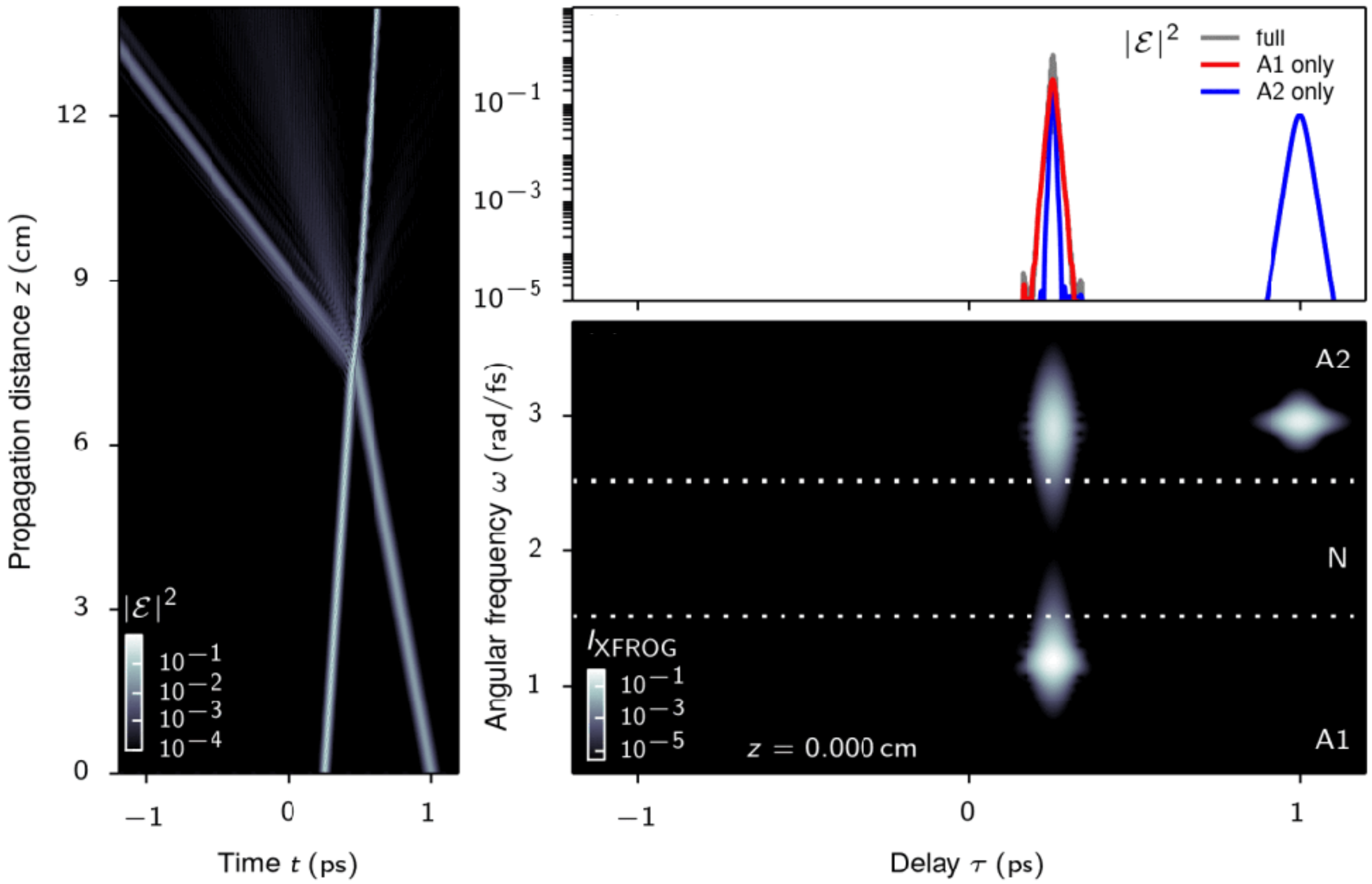
mutual interaction disabled

➡ constituent pulses are no solitons

- co-propagating pulses mutually sustain their shape
- limits of mutual binding can be explained by simple models

[Melchert, Willms, Morgner, Babushkin, Demircan; Sci. Rep. 11 (2021) 11190]

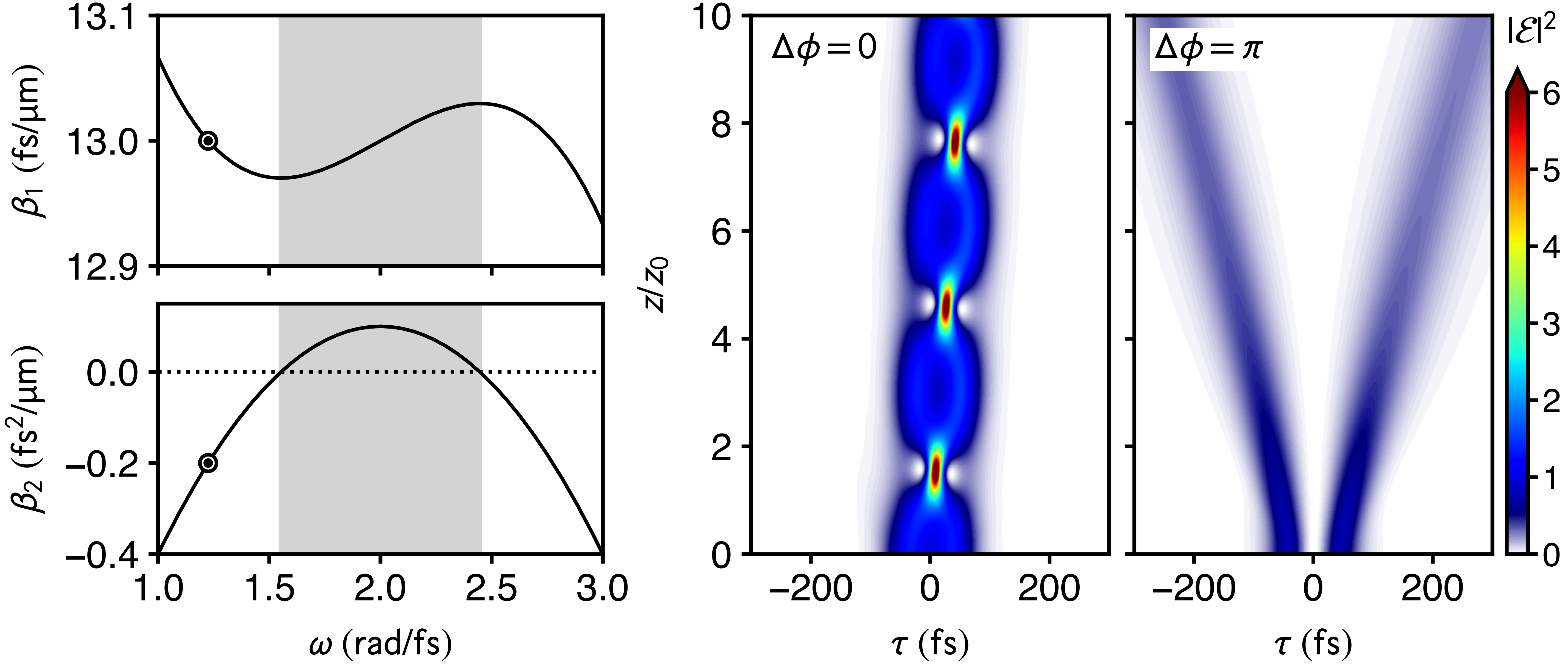
Robustness against perturbation



Dynamical evolution of fundamental solitons

- Initially overlapping solitons
  - solitons have same center frequency
  - both are initially group-velocity matched
  - ➡ phase dependent soliton-soliton interaction

$$E_0(t) = \mathrm{Re}\left[\frac{A_1\, e^{-i\omega_1 t}}{\cosh[(t+\delta)/t_1]} + \frac{A_2\, e^{-i(\omega_2 t + \Delta\phi)}}{\cosh[(t-\delta)/t_2]}\right]$$

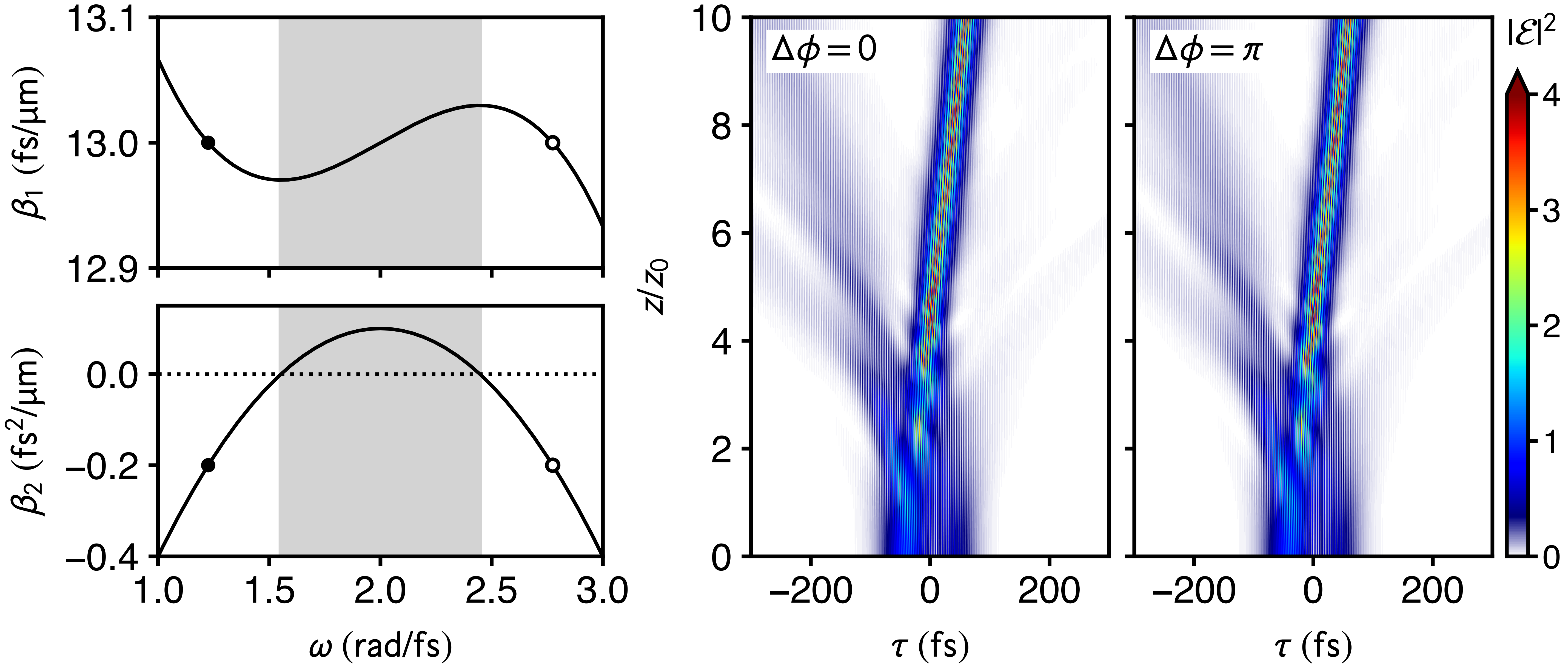


[Melchert, Demircan; Opt. Lett. 46 (2021) 5603]

Dynamical evolution of fundamental solitons

- Initially overlapping solitons
  - solitons have vast frequency gap
  - both are initially group-velocity matched
  - ➡ dynamics dominated by **incoherent** interaction between solitons

$$E_0(t) = \mathrm{Re}\left[\frac{A_1\, e^{-i\omega_1 t}}{\cosh[(t+\delta)/t_1]} + \frac{A_2\, e^{-i(\omega_2 t+\Delta\phi)}}{\cosh[(t-\delta)/t_2]}\right]$$



[Melchert, Demircan; Opt. Lett. 46 (2021) 5603]

A simplified theoretical model

- Assumptions and approximation steps
  - introduce reference frequency + shift to moving frame
  - approximate dynamics by two NSEs coupled through cross-phase modulation (XPM)
  - ➡ models **incoherently** interacting pulses

$$i\partial_z u_1 - \frac{\beta_2'}{2}\partial_\tau^2 u_1 + \gamma' \left(|u_1|^2 + 2|u_2|^2\right) u_1 = 0$$
$$i\partial_z u_2 - \frac{\beta_2''}{2}\partial_\tau^2 u_2 + \gamma'' \left(|u_2|^2 + 2|u_1|^2\right) u_2 = 0$$

$$\omega_1 = \omega_{\mathrm{M1}} \qquad \omega_2 = \omega_{\mathrm{M2}}$$
$$\beta_2' = \beta_2(\omega_1) = -2\beta_2 \qquad \beta_2'' = \beta_2(\omega_2) = -2\beta_2$$
$$\gamma' = \gamma(\omega_1) \qquad \gamma'' = \gamma(\omega_2)$$
$$\tau = t - \beta_1(\omega_0) z$$

- Restricting to pulses of same width yields ***effectively decoupled*** equations

$$u_n(z,\tau) = N_n\, A_n\, \mathrm{sech}(\tau/t_0)\, e^{i\kappa_n z}, \quad n \in (1,2)$$

N_n = deviation from fundamental soliton

κ_n = suitable wavenumber

$$i\partial_z u_1 - \frac{\beta_2'}{2}\partial_\tau^2 u_1 + \Gamma'|u_1|^2 u_1 = 0,$$
$$i\partial_z u_2 - \frac{\beta_2''}{2}\partial_\tau^2 u_2 + \Gamma''|u_2|^2 u_2 = 0$$

$$\Gamma' = \gamma'(1 + 2\alpha N_2^2 N_1^{-2})$$
$$\Gamma'' = \gamma''(1 + 2\alpha^{-1} N_1^2 N_2^{-2})$$
$$\alpha = \frac{|\beta_2''|\gamma'}{|\beta_2'|\gamma''}$$

[Melchert, Demircan; Opt. Lett. 46 (2021) 5603]

A simplified theoretical model

- Closed form solutions describing two-color ***soliton*** pairs

$$u_n(z,\tau) = N_n\, A_n\, \mathrm{sech}(\tau/t_0)\, e^{i\kappa_n z}, \quad n \in (1,2)$$

$$N_1 = \sqrt{\frac{2\alpha-1}{3}} \qquad N_2 = \sqrt{\frac{2\alpha^{-1}-1}{3}} \qquad \alpha = \frac{|\beta_2''|\gamma'}{|\beta_2'|\gamma''}$$

$$A_1 = \sqrt{2\beta_2/\gamma(\omega_1)}/t_0 \qquad A_2 = \sqrt{2\beta_2/\gamma(\omega_2)}/t_0$$

solutions only for

$$\frac{1}{2} < \alpha < 2$$

- Two-color ***soliton*** pairs
  - each subpulse specifies a soliton solution of a standard NSE
  - they can only persist conjointly as a bonding unit
  - effect of binding partner is to modify nonlinear coefficient

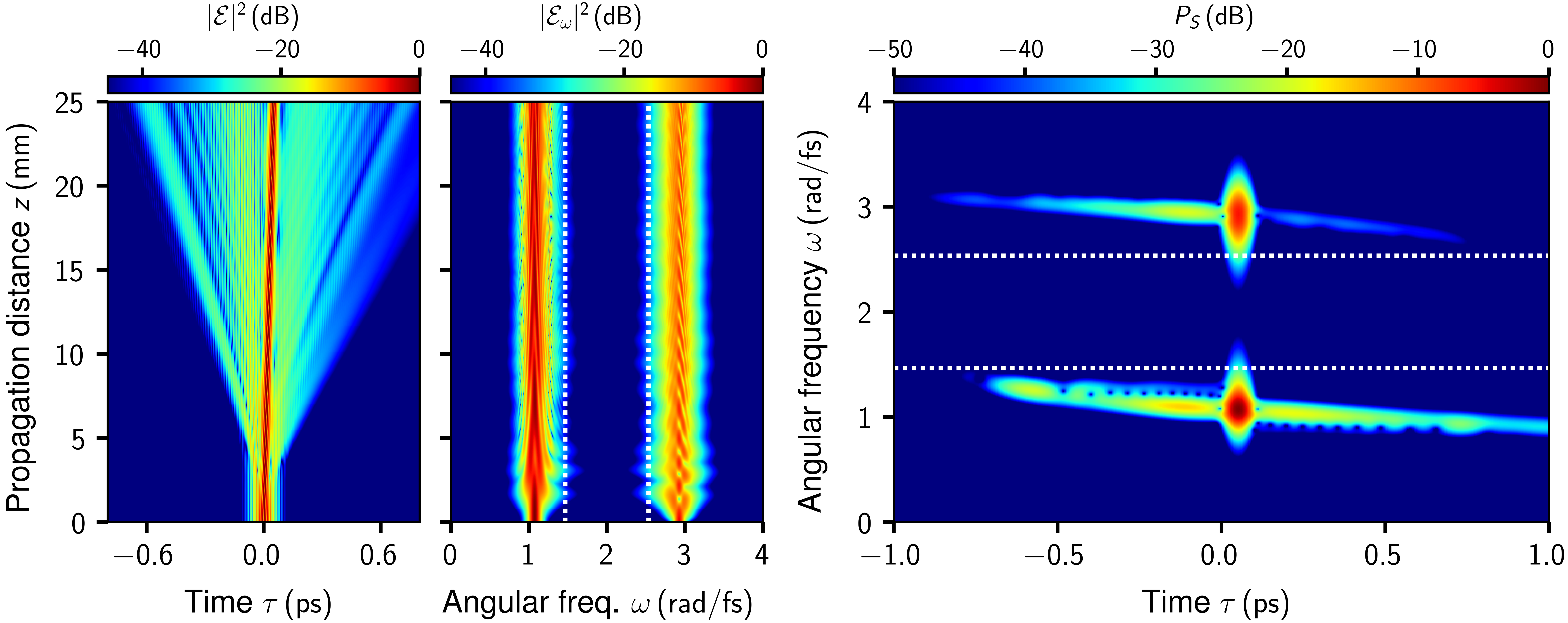
- Limiting case of equivalent subpulses (generalized dispersion Kerr solitons) $\beta_2' = \beta_2'' = -2\beta_2$, $\gamma' = \gamma''$
  [Tam *et al.*; Phys. Rev. A 101 (2020) 043822]
  - ***fundamental metasoliton*** is obtained without complicated multi-scales analysis*

$$F = u_1 + u_2 = \sqrt{\frac{8\beta_2}{3\gamma t_0^2}}\mathrm{sech}(\tau/t_0)e^{i\kappa z}, \quad \text{with} \quad \kappa = \frac{\beta_2}{t_0^2}$$

* thorough comparison in Supp. Mat. of: **[Melchert, Demircan; Opt. Lett. 46 (2021) 5603]**

Immediate consequence for our theoretical studies

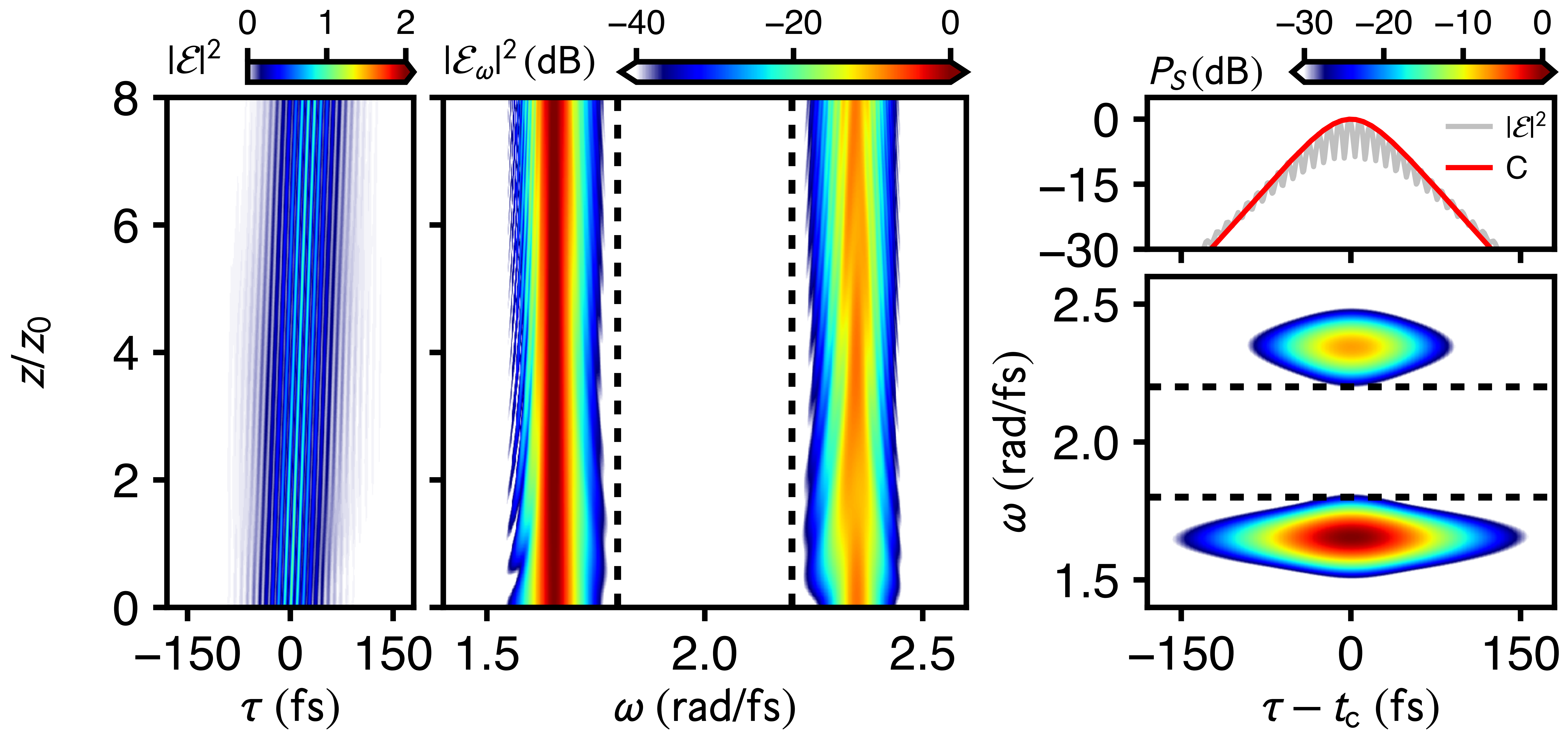
- Instead of generating molecules this way:



[Melchert, Willms, Morgner, Babushkin, Demircan; Sci. Rep. 11 (2021) 11190]

Immediate consequence for our theoretical studies

- We can now directly initialize them:

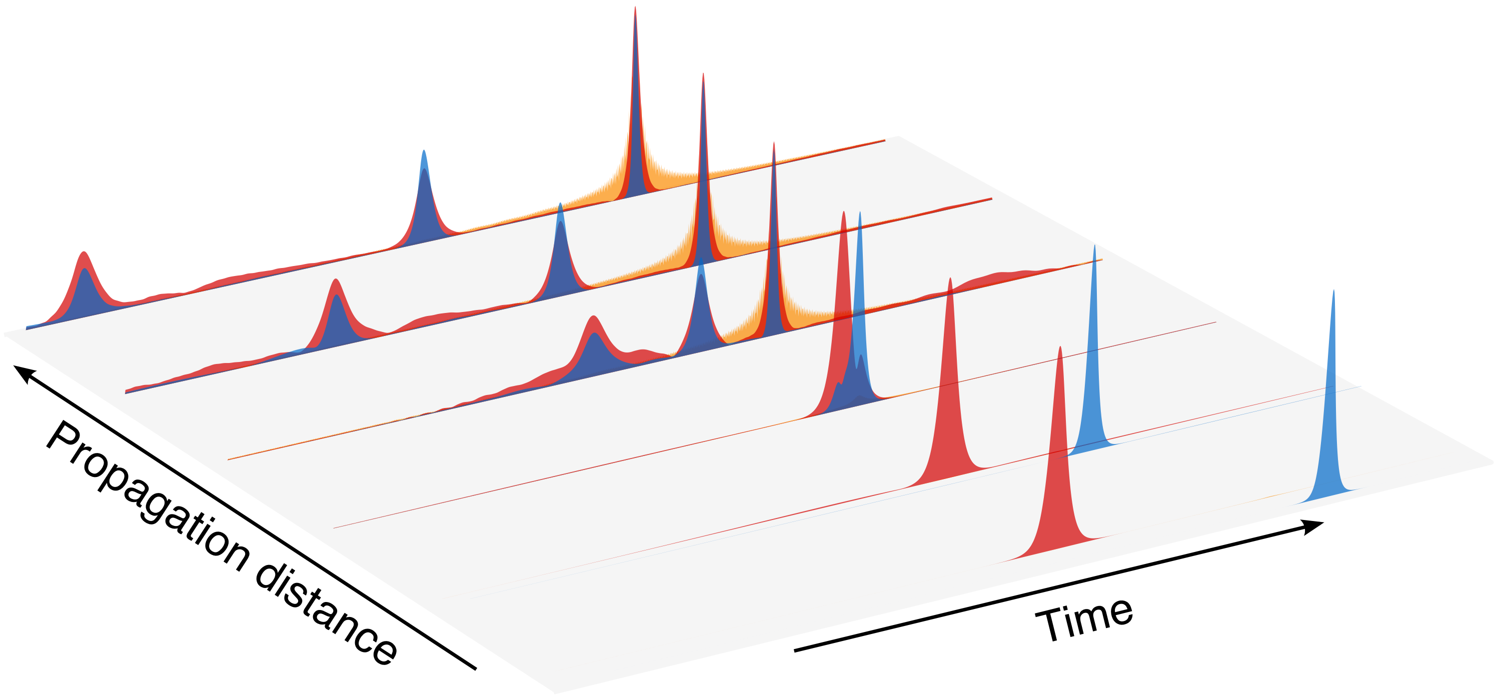


[Melchert, Demircan; Opt. Lett. 46 (2021) 5603]

Summary

Modeling pulse propagation in nonlinear waveguides

- forward model for the analytic signal
- nonlinear Schrödinger equation
- generalized nonlinear Schrödinger equation



New phenomena involving two-frequency pulse compounds

- enabled by group-velocity matching across a vast frequency gap
- trapped states
- molecule-like bound states

DFG Deutsche Forschungsgemeinschaft
(EXC 2122, projectID 390833453)